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ABSTRACT

Planning for a computer-based personnel job opportunities system for the Air Force led to a requirement for a procedure to generate a "payoff" or "value" of the assignment of each person to each possible job. This report discusses three methods of weighting different information to form a single indicator "payoff" or "value," explicit weighting and two implicit weighting methods--policy-capturing and policy-specifying. The two implicit weighting methods are combined into a more comprehensive method referred to as policy-development. The policy-capturing process presents a series of decision situations to one (or more) policy makers and the policy maker assigns to each situation a number which reflects the "value" or "payoff." Then a mathematical model is derived by obtaining the regression equation that best predicts the policy maker's judgments. Policy-specifying, which is the main focus in this report, does not depend on a sample of actual judgments to determine the regression weights, but attempts to translate into mathematical form a policy maker's more global statements about the general properties that the model should have. The mixing of policy-capturing and policy-specifying leads to a process called policy-development. (Author)

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CREATING MATHEMATICAL MODELS OF JUDGMENT PROCESSES:

FROM POLICY-CAPTURING TO POLICY-SPECIFYING

By

Joe H. Ward, Jr.

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OCCUPATION AND MANPOWER RESEARCH DIVISION
Brooks Air Force Base, Texas 78235

August 1977
Final Report for Period 1 July 1975 – 1 July 1976

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Planning for a computer-based personnel job opportunities system for the Air Force led to a requirement for a procedure to generate a "payoff" or "value" of the assignment of each person to each possible job. This report discusses three methods of weighting different information to form a single indicator of "payoff" or "value," explicit weighting and two implicit weighting methods – policy-capturing and policy-specifying. The two implicit weighting methods are combined into a more comprehensive method referred to as policy-development.

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The mixing of policy-capturing and policy-specifying leads to a process called policy-development.

PREFACE

During 1974–1976, meetings were held between personnel of Air Force Human Resources Laboratory (AFHRL), Air Force Recruiting Service and Air Force Military Personnel Center to discuss the information to be included in the payoff function to reflect the value to the Air Force of assigning applicants to various Air Force jobs. While many people made valuable input, major contributions were provided by Mr. Tom Beatty, Mr. Bob Cantu, Capt Harry Haltman, Major Gordon Markham, LtCol Jack Tillman, and Capt Tom Van Sweringen.

Within AFHRL Occupation and Manpower Research Division, significant contributions were made by Dr. Raymond Christal, Major William Hendrix, Capt Don Haney, Mr. Manuel Pina, A1C Henry McLin, Sgt Bill Solomon and Capt Mike Hawkins. Capt Don Haney's assistance in checking the detailed development of the policy-specified models greatly accelerated their development. The policy-specifying FORTRAN program, included in Appendix I, was developed by Sgt Bill Solomon. Mr. Manuel Pina, who was in charge of developing the computer-based research and demonstration program, provided extensive tests of the payoff system that led to numerous improvements. Mrs. Helen Widner's careful draft typing was essential to completion of the report.

The development of these models was made possible by on-line access to the Burroughs 6700 at the U.S. Air Force Academy, Colorado and to the UNIVAC computer of AFHRL, Lackland AFB, Texas.

This research was completed under work unit 20770401, Development of an Advanced Pre-Enlistment Person-Job-Match System for Air Force Enlistees for use in the All-Volunteer Environment. Preliminary research was conducted under work unit 20770308, Research in Support of Recruiting Service Operations.

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CREATING MATHEMATICAL MODELS OF JUDGMENT PROCESSES: FROM POLICY-CAPTURING TO POLICY-SPECIFYING

I. INTRODUCTION

Following feasibility studies (Hawkins, Crow, & Haltman, 1974) and demonstrations (Ward & Haltman, 1975) a computer-based system was being developed for selecting and classifying personnel who are considering enlisting in the Air Force. This system is a part of the Air Force's Advanced Personnel Data System Procurement Management Information System (APDS-PROMIS). Within this system there is a need for numerical values that will indicate the "value" or "payoff" to the Air Force or recruiting a particular person for a particular job at a specific time. The value to be used is sometimes determined by judgment of one or more policy makers. Frequently this value is obtained by combining several different types of information into a weighted composite to produce a numerical indicator of the policy maker's judgment of "value."

One method of weighting is to have the policy maker explicitly provide the numerical weights to be used with the different types of information to form the composite (explicit-weighting). Explicit-weighting is satisfactory in some situations. It is usually difficult, however, to choose the proper multiplier values to form the composite values that adequately express the worth of a person on a job.

The difficulties encountered with explicit-weighting have led to a second method – policy-capturing – which involves implicit determination of the numerical weights. In the policy-capturing process the policy maker observes various decision situations and assigns a number to reflect the "value" of each situation. For example, a policy maker may be presented a series of information profiles each of which reflects important data about an Air Force applicant, such as aptitude test scores, difficulty of the job being considered, applicant preferences, etc. The policy maker assigns a number to each profile, which reflects the value to the Air Force of assigning the applicant to a particular job. Then the weights are computed – by least squares regression – that best predict the judged values from variables derived from the information available about each decision situation. Some examples of policy-capturing applications have been described in the following publications: Black (1973); Christal (1968a, 1968b); Gott (1974); Gooch (1972); Jones, Mannis, Martin, Summers, and Wagner (1976); Koplyay (1970); Koplyay, Albert, and Black (1976); Mullins and Usdin (1970); Ward and Davis (1963).

During the early stages of development of the APDS-PROMIS system many discussions took place concerning various approaches to obtaining an expression of value for the different person-job assignments. It seemed appropriate to create the values of the assignments by means of the policy-capturing process. However, there was considerable hesitancy by everyone concerned to launch into policy-capturing. Some other approach seemed to be required.

It is difficult to identify all of the reasons for not carrying through the policy-capturing process. However, the major difficulties seemed to be related to the existence of two different types of information to be weighted into the value composite. The first type might be called *management-related*, such as "filling of quotas" and "maintaining minority balance," and the second type, *quality-of-assignments-related*, such as "matching a person to a job in which he will perform well and be satisfied." The two types of information contributed to another problem in policy-capturing – "who will be the panel of policy makers?" There might be judges who can adequately combine the management-related information and there might be judges who can handle the quality-of-assignments-related information. But it was felt that it would be difficult to identify policy makers who could appropriately combine both types of variables into an acceptable policy through the policy-capturing process. Furthermore, it seemed likely that a mixed panel of policy makers (management-related vs. quality-of-assignments-related) would not yield an acceptable model through policy-capturing. And failure to arrive at a policy model could strain relations among the policy makers.

Time was getting short. A value generator was needed. It was decided that a starting policy model should be created reflecting as well as possible the "expressions of policy" that had emerged through the numerous discussions among personnel managers and researchers. Output values from this starting model would be displayed and used in a demonstration of the PROMIS assignment procedure. Both personnel managers and researchers could examine the generated values and provide comments to the model makers for revising the model. This strategy led to the development of another implicit-weighting approach – policy-specifying. This third method provides another way of reflecting a policy maker's value judgments in a mathematical model without requiring explicit-weighting and without the more lengthy policy-capturing process.

The process of policy-specifying requires a translation into a mathematical model of the policy makers' general statements about the way information is combined to generate values. After a model is created and the results are displayed to the policy makers, a new model is evolved to reflect the policy more precisely. The process of creating new models with new properties continues until the outputs are acceptable to the policy makers.

Repeatedly creating new models for the generation of the PROMIS payoffs was time-consuming and the long cycle time required to display the output from a new model threatened an unacceptable delay in implementing the APDS-PROMIS system.

In order to reduce the time required to create new models for examination by policy makers, it was decided to develop a model generating system. This model generator is controlled by parameter settings through an interactive computer program. It allows the model developer to greatly reduce the time required to bring models into alignment with desired policy.

The power of this model generator was realized during the last two weeks before nationwide operational implementation of APDS-PROMIS. Policy makers took a last minute closer look at the existing (soon to be operational) payoff generator. They were not happy. Using a remote terminal connected by phone from the policy maker's office at Randolph AFB to the AFHRL UNIVAC 1108 Computer at Lackland AFB new models were quickly examined and tested in the operational system. An acceptable payoff generator was created and APDS-PROMIS went operational on schedule.

The general models developed in this report for policy-specifying in the personnel assignment problem can be applied to many other situations by varying the parameters of the models to obtain the desired characteristics. When these model forms are not applicable, the appropriate models can be developed by imposing the restrictions systematically as shown in Appendixes A through G of this report and in *Introduction to Linear Models* (Ward & Jennings, 1973) and *Applied Multiple Linear Regression* (Bottenberg & Ward, 1963).

The remainder of this report focuses in Section II on the two implicit-weighting methods mentioned above – policy-capturing and policy-specifying – and their combination referred to as policy-development. Section III contains several specific examples of policy-specifying that arose during the development of the person-job-match component of APDS-PROMIS. Section IV contains a description of the model generator and its application to the specific examples in Section III. The detailed developments of the models and the FORTRAN program for the model generator are included as appendixes.

II. IMPLICIT WEIGHTING IN POLICY DEVELOPMENT

The weights derived from a policy-development process can be viewed on an implicit-weighting continuum. On one end of the continuum are the implicit weights derived from the policy-capturing process, and at the other extreme are the implicit weights derived from the policy-specifying process. This section focuses first on the implicit weights derived from policy-capturing and policy-specifying. This is followed by a discussion of the implicit weights from general policy-development which combines both capturing and specifying.

Policy-Capturing Weights

Extensive discussions of the policy-capturing process are available in the references previously cited. The focus here is on the regression weights obtained from policy-capturing. The implicit weights obtained from policy-capturing are obtained by solving for coefficients that when used to form a composite of predictor information will best predict the judgments.

Specifically, let

- Y = a vector of judged values of dimension n. These n judgments are obtained from a policy maker, who examines each decision situation and assigns a value to be associated with the situation.
- U = the unit vector of dimension n, with all elements equal 1.
- $X^{(j)}$ = the jth predictor vector, of dimension n generated from the information associated with the decision situations. $j = 1, \ldots, k$. Assume that $n > k + 1$.
- E = the error vector of dimension n.
- a_j = the unknown weights associated with $X^{(j)}$ to be implicitly determined to minimize the error sum of squares. $j = 1, \ldots, k$.
- a_o = the unknown weight associated with U.

Then the prediction model may be written

$$Y = a_o U + a_1 X^{(1)} + a_2 X^{(2)} + \ldots + a_j X^{(j)} + \ldots + a_k X^{(k)} + E^{(1)} \qquad (1)$$

and the least squares weights can be used to predict the Y values for new situations.

While the policy-capturing process has been quite useful for modeling judgments, another procedure for obtaining weights may have advantages in some situations.

Policy-Specifying Weights

Policy-capturing requires a set of judgments (Y values) associated with n decision situations to obtain the implicit weights. However, in the policy-specifying process the weights are determined without empirically obtained judgments (Y values) by stating desired properties of and relations among the predicted values in sufficient detail that the numerical weights become known.

Specifically let

- b_j = the unknown weights to be determined by policy-specifying (corresponding to a_j in policy-capturing above). $j = 1, \ldots, k$.
- b_o = an unknown constant (corresponding to a_o)
- x_j = variables corresponding to the predictor vectors above. These are not *vectors* of data but are variables which when given a set of weights b_j and b_o and a set of values for x_j will yield a composite value y.

Then we have the starting function

$$y = b_o + b_1 x_1 + b_2 x_2 + \ldots + b_j x_j + \ldots + b_k x_k \qquad (2)$$

Prior to the policy-specifying process the range of values for $x_1, x_2, \ldots, x_k$ are known but the b_j and b_o values are not known. Policy-specifying proceeds by stating restrictive relations among the predicted values for various values of x_j. These policy statements result in restrictions on the values of b_j and b_o so that the numerical values of the weights can be determined. Specification is completed when $k + 1$ independent restrictions are imposed. Once the values of b_j and b_o are known then predicted values, y, can be calculated for any values x_j.

This process of policy-specifying is the same as that required for imposing restrictions on linear statistical models to test hypotheses. The procedures described by Bottenberg and Ward (1963) and Ward and Jennings (1973) can be directly applied in determining the weights implied by the policy-specifying process.

This report focuses on specific examples of policy-specifying and general forms which might be applicable to new situations.

Policy-Development Weights

Policy-capturing and policy-specifying can be combined to form a general process of policy-development. A particular decision maker may start by specifying several properties about relations among the predicted values in the function (2). Whereas policy-specifying resulted in $k + 1$ restrictions on the $k + 1$ weights, b_j and b_o, the expression of desired properties may result in only $r < k + 1$ restrictions on the b_j and b_o values.

Then imposing these r restrictions on the starting model (2) results in a restricted model

$$y_r = c_o + c_1 z_1 + c_2 z_2 + \ldots + c_j z_j + \ldots + c_{k-r} z_{k-r} \qquad (3)$$

where

z_i = new variables resulting from imposing the r restrictions.

Each z_i variable is a linear combination of the x_i variables. Now since there are still $k + 1 - r$ unknown weights c_j and c_o to be computed it would be possible to use policy-capturing to find the c_j values. The decision maker could provide, for each of n $[n > (k + 1 - r)]$ decision situations, y_i $(i = 1, \ldots, n)$ values associated with various profiles of information about the different situations. Then the least squares values of c_j can be computed for the model

$$Y = c_o U + c_1 Z^{(1)} + c_2 Z^{(2)} + \ldots + c_j Z^{(j)} + \ldots + c_{k-r} Z^{(k-r)} + E^{(2)} \qquad (4)$$

where

Y = a vector of judged values of dimension n.

$Z^{(j)}$ = the jth predictor vector, of dimension n formed as linear combinations of the predictor vectors $X^{(j)}$ generated from information associated with the decision situations.

Having computed the least squares values for c_j and c_o the weighting system now produces values that both reflect the policy restrictions imposed by the policy-specifying process and the best fit to the empirical judgments.

The remainder of this report will concentrate on policy-specifying procedures. The next section will provide several examples of the use of policy-specifying. This will be followed by generalizations which are applicable to new situations.

III. POLICY-SPECIFYING EXAMPLES

This section contains several examples of policy-specifying that arose during the development of the person-job-match component of APDS-PROMIS. Focus in these examples is on description of the policy and the model that results from translating the natural language policy statements into mathematical form. Model specifying details which are required to develop the models are presented in the appendixes. The mathematical restrictions, imposed in the appendixes to create the various models, are selected to both approximate the policy statements and to allow for easy generation and control of the models.

Example 1: Value to Air Force as Function of Aptitude and Job Difficulty

The most important example arising in PROMIS is the expression of policy about the "value to the Air Force" of assigning a particular person to a particular job. While there are several variables (or components) that contribute to this expression of worth, the most important component involves the expression of value of the person-job assignment as a function of only two basic properties – aptitude of the person and difficulty of the job.

The policy maker indicated the following desired properties of his values:

1. The range of composite numbers y to express "value" would be from 0 to 100.

2. A value of 100 would be assigned when a person with maximum aptitude (a = 95) is assigned to a job of maximum difficulty (d = 100).

3. Values of 0 would be assigned when a person's aptitude is about 15 or 20 points below the difficulty measure of the job.

4. A value of 15 would be assigned when a person with minimum aptitude (a = 40) is assigned to a job of minimum difficulty (d = 40).

5. A value of 35 would be assigned when a person with maximum aptitude (a = 95) is assigned to a job of minimum difficulty (d = 40).

6. The values for a person with maximum aptitude (a = 95) will start at y = 35 when a = 40 and increase gradually, reaching the maximum value y = 100 at d = 100. This policy statement is sketched in Figure 1.

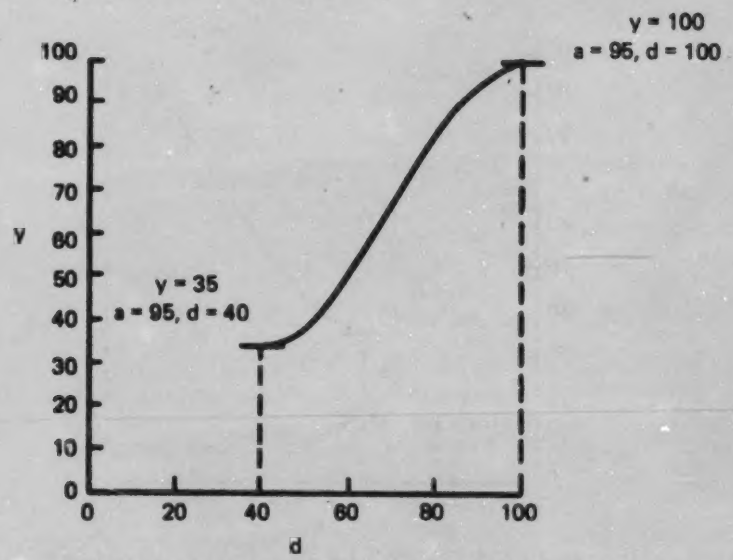


Figure 1. Y (payoff) = function of d at a = 95.

7. A person of minimum aptitude (a = 40) will have a maximum value (y = 15) when assigned to a minimum difficulty job (d = 40). The values for this person will start at y = 15 when a = 40 and decrease gradually to y = 0 when the job difficulty is about 60. This can be sketched as shown in Figure 2. The combination of policies 6 and 7 are shown in Figure 3.

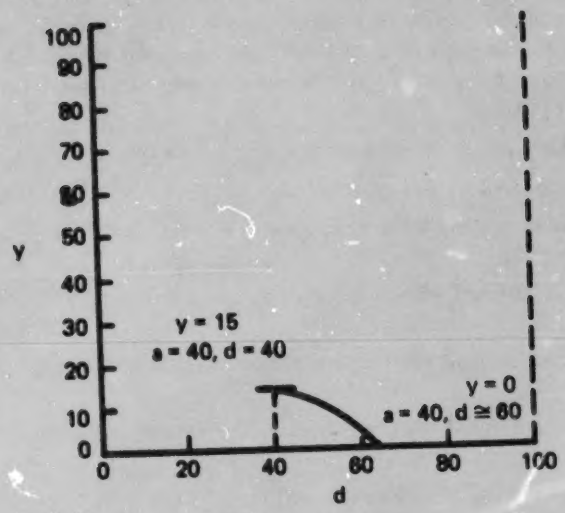


Figure 2. Y (payoff) = function of d at a = 40.

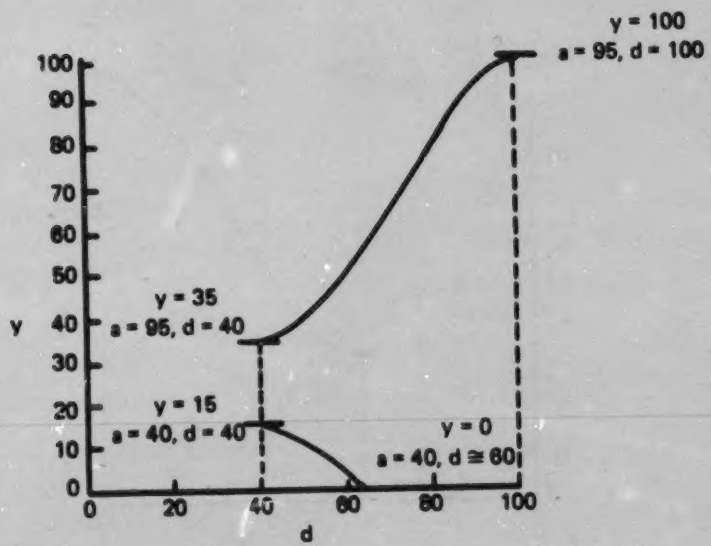


Figure 3. Y (payoff) = function of d at a = 40 and a = 95.

8. Persons between the extreme aptitudes ($40 < a < 95$) will have their maximum values about when the aptitude value is approximately equal to or slightly greater than the difficulty measure. This policy can be combined with statements 6 and 7 to give the following sketch shown in Figure 4.

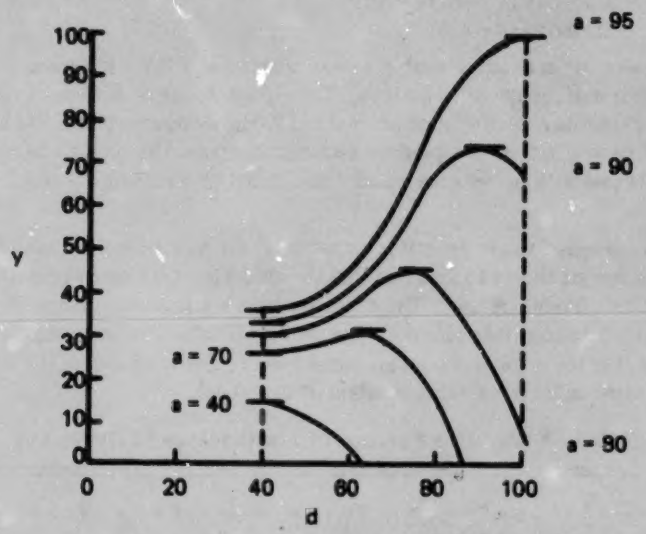


Figure 4. Y (payoff) = function of d at a = 40, a = 70, a = 80, a = 90 and a = 95.

9. The policy maker stated that for a job of minimum difficulty (d = 40) the amount of change in value per unit change in aptitude would be constant and the values should increase only slightly. As a moves from 40 to 95, y changes from 15 to 35.

10. As the job difficulty increases, the amount of change in value per unit change in aptitude increases rapidly. Statements 9 and 10 are sketched as shown in Figure 5.

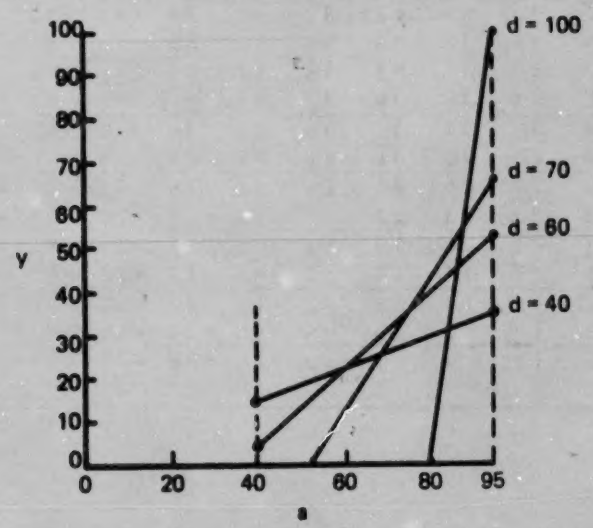


Figure 5. Y (payoff) = function of a at d = 40, d = 60, d = 70 and d = 100.

Following the procedures detailed in Appendix A, the policy specified model becomes

$$y = 35 + .3636(a-95) + .05417(d-40)^2 + .00001136(a-95)(d-40)^3 + .0009848(a-95)(d-40)^2 - .0006019(d-40)^3$$

The highest power for aptitude (variable a) was selected as 1 since statement 9 indicates a constant change in value (y) per unit change in aptitude (a). The highest power of difficulty (variable d) was selected as small as possible (3) while reflecting statements 6 and 7. Higher powers of d could be used and presented to policy makers if power 3 does not produce desired outcomes. The product between aptitude (a) and difficulty (d) reflects interactions between a and d as implied in statements 6 and 7, and in statements 9 and 10.

Table 1 contains selected values generated from the above policy-specified model. Observe that values outside the critical range are shown to illustrate that the model has the desired properties. Only the positive values generated by this model are used. The eligibility tests applied prior to use of this model eliminate aptitude-difficulty combinations that produce negative payoff values. Observe that all slopes are zero at d (difficulty) = 40 and that the ridge of maximum values moves from 15 at a = 40, d = 40 to 100 at a = 95, d = 100. Figure 6 is a three-dimensional representation of the model.

Table 1. Y (Payoff) = Function of a (aptitude) and d (difficulty)

```
Y =  35 + [< .3636+00>*(A -  95)** 1] + [< .5417-01>*(D -  40)** 2]
        + [< .1136-04>*[(A -  95)** 1]*[(D -  40)** 3]]
        + [< .9848-03>*[(A -  95)** 1]*[(D -  40)** 2]]
        + [<-.6019-03>*(D -  40)** 3]
```

		D I F												
	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105
100	38	37	38	42	48	56	65	75	86	96	107	116	125	132	137
95	36	35	36	40	45	52	59	67	76	83	90	95	99	100	99
90	34	33	34	37	42	48	54	60	65	70	73	74	73	68	60
85	33	31	32	35	39	43	48	52	55	56	56	53	46	35	22
80	31	30	30	33	36	39	42	44	45	43	39	31	20	5	-16
75	29	28	28	30	33	35	36	36	34	30	22	10	-6	-27	-54
A 70	27	26	27	28	30	31	31	29	24	16	5	-11	-32	-59	-93
P 65	25	24	25	26	26	26	25	21	14	3	-12	-32	-58	-91	-131
T 60	23	22	23	23	23	22	19	13	4	-10	-29	-53	-84	-123	-169
55	21	20	21	21	21	20	18	13	5	-7	-24	-46	-75	-111	-155	-207
50	19	19	19	19	19	17	14	7	-3	-17	-37	-63	-96	-137	-186	-245
45	17	17	17	16	14	9	2	-10	-27	-50	-80	-117	-163	-218	-284
40	15	15	15	14	11	5	-4	-18	-38	-64	-97	-138	-189	-250	-322
35	13	13	13	11	8	1	-10	-26	-48	-77	-114	-160	-215	-282	-360

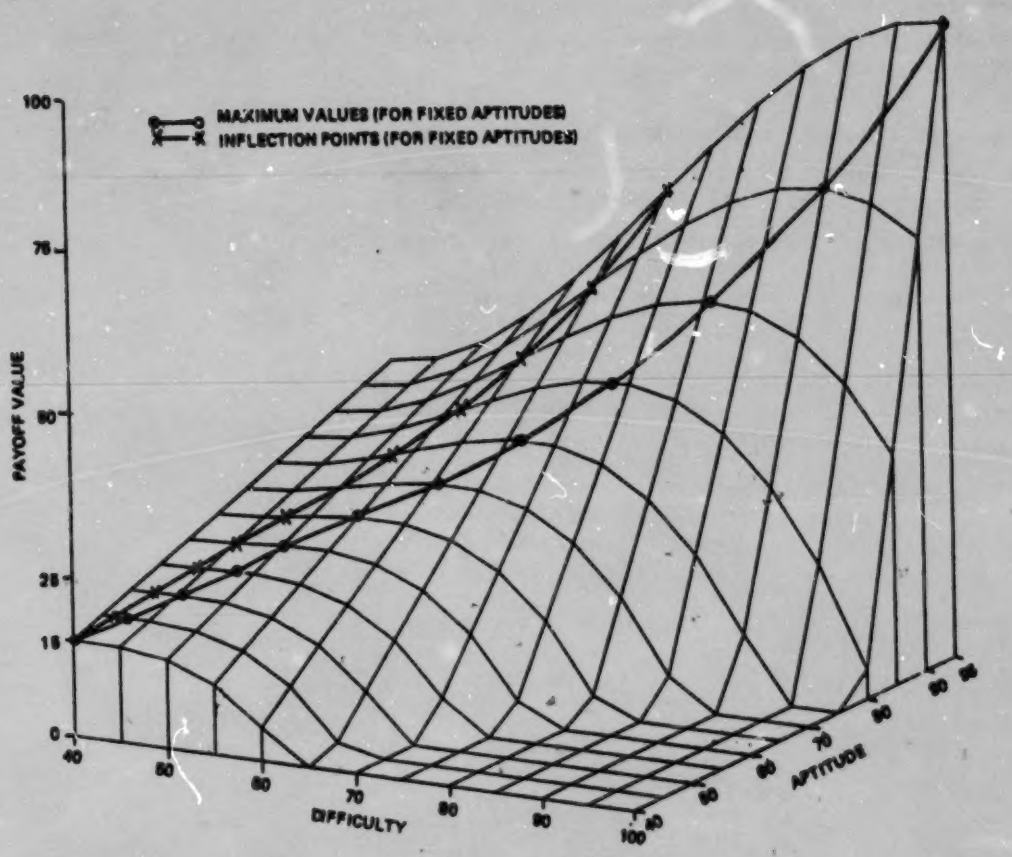


Figure 6. Aptitude-Difficulty component.

Example 2: Value to Air Force as Function of Time Used and Fraction of Fill

Another component that contributes to overall value of assigning a particular person to a job is related to the particular job's importance, the time used, and the fraction of the quota already filled.

It is assumed in this example that when a job quota becomes open there are 180 days left to fill the quota.

The policy maker stated the following characteristics of his values:

1. The range of composite numbers y to express "value" is from 0 to 100.

2. There should be an importance number, k, (between 0 and 100) that can be used to emphasize certain jobs more strongly than others independent of the time used and fraction of fill.

3. A value of y close to 100 would be assigned for a completely unfilled quota with almost no time left (near time, t = 180 days and fraction of quota filled, f = .0).

4. A value close to 0 would be assigned for an almost completely filled job with maximum time left (near t = 0 days, f = 1.0).

5. The importance number, k, associated with the job would be the value of y when time used, t = 0 days and fraction of quota filled, f = .0.

6. The value of y approaches k, the job-importance number, when the quota is close to being filled with almost no time left (near t = 180 and f = 1.0).

7. For any fraction of fill, the amount of change in value per unit change in time is constant; however, the constant change might be different for different values of fill.

8. For any time, the amount of change in value per unit change in fraction fill is constant; however, the constant change might be different for different values of time.

Translating these policy statements into a model (details given in Appnedix B) yields

$$y = k + \frac{(100 - k)}{180}t + (-k)f + \frac{(2k - 100)}{180}(tf) \qquad (2)$$

Notice that the highest power for both t and f is one. This reflects statements 7 and 8. Also the product between t and f reflects the interactions implied in statements 7 and 8.

For example, consider a low priority job with k = 25.

Then

$$y = 25 + \frac{75}{180}t - 25f - \frac{50}{180}(tf)$$

$$= 25 + .4167(t) - 25f - .2778(tf)$$

Two sketches of this policy are shown in Figures 7 and 8.

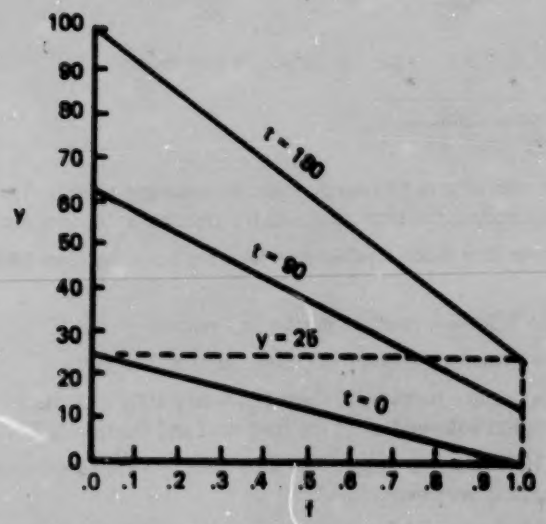


Figure 7. Y (payoff) = function of f for k = 25 at t = 0, t = 90, t = 180.

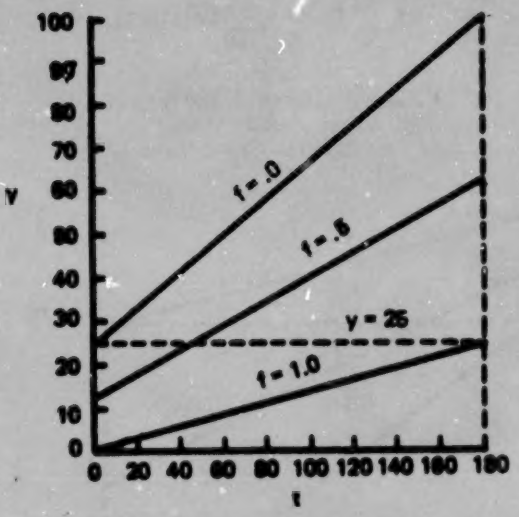


Figure 8. **Y (payoff) = function of t for k = 25 at f = .0, f = .5, f = 1.0.**

Values of y for selected combinations of t and f are shown in Table 2.

Table 2. **Y (Payoff) = Function of t (time used) and f (fraction fill) for k = 25**

Y = 25 + [< .4167+00>*(T - 0)** 1] + [<- 25.00+00>*(F - 0)** 1]
+ [<-.2778-00>*[(T - 0)** 1]*[(F - 0)** 1]]

TIME	FILL .0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	93	85	78	70	63	55	48	40	33	25
165	94	87	80	73	65	58	51	44	37	30	23
150	87	81	74	68	61	54	48	41	34	28	21
135	81	75	69	63	56	50	44	37	31	25	19
120	75	69	63	58	52	46	40	34	28	22	17
105	69	63	58	52	47	42	36	31	25	20	15
90	62	58	53	48	43	38	33	29	23	18	13
75	56	52	47	43	38	33	29	24	20	15	10
60	50	46	42	38	33	29	25	21	17	12	8
45	44	40	36	33	29	25	21	17	14	10	6
30	37	34	31	27	24	21	17	14	11	7	4
15	31	28	25	23	20	17	14	11	8	5	2
0	25	23	20	18	15	13	10	8	5	3	0

A high priority job, for example k = 75 would give

$$y = 75 + \frac{25}{180}t - 75f + \frac{50}{180}(tf)$$

$$= 75 + .1389(t) - 75(f) + .2778(tf)$$

Two sketches of the high priority policy are shown in Figures 9 and 10.

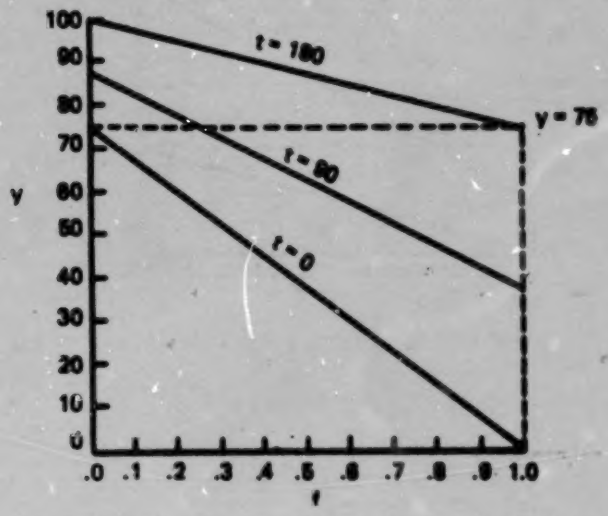


Figure 9. Y (payoff) = function of f for k =75 at t = 0, t = 90, t = 180.

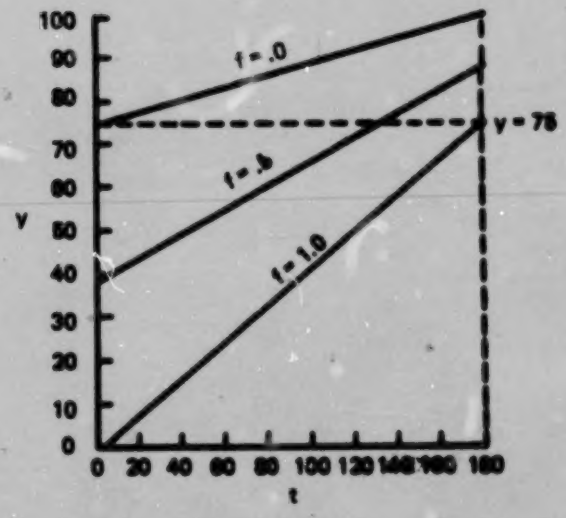


Figure 10. Y (payoff) = function of t for k = 75 at f = .0, f = .5, f = 1.0.

Values of y for selected combinations of t and f are shown in Table 3.

Table 3. **Y (Payoff) = Function of t (time used) and f (fraction fill) for k = 75**

```
Y =   75 + L< .1389+00>*(T -   0)** 1] + 1<- 75.00+00>*(F -   0)** 1]
        + L< .2778-00>*L(T -   0)** 1]*L(F -   0)** 1]]
```

		FILL										
		.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
	180	100	97	95	92	90	87	85	82	80	77	75
	165	98	95	92	89	86	83	80	77	75	72	69
	150	96	92	89	86	82	79	76	72	69	66	62
	135	94	90	86	82	79	75	71	67	64	60	56
T	120	92	87	83	79	75	71	67	62	58	54	50
I	105	90	85	80	76	71	67	62	57	53	48	44
M	90	87	82	77	72	67	62	57	52	47	42	37
E	75	85	80	75	69	64	58	53	47	42	37	31
	60	83	77	72	66	60	54	48	42	37	31	25
	45	81	75	69	62	56	50	44	37	31	25	19
	30	79	72	66	59	52	46	39	32	26	19	12
	15	77	70	63	56	49	42	35	27	20	13	6
	0	75	68	60	53	45	38	30	23	15	8	0

Example 3: Value to Air Force as Function of Fill and Known Goal

Another type of value component relates to the fraction of quota fill compared to a specific goal. This policy could be developed to reflect the value of an assignment as it relates to the fraction of quota filled by a minority group compared to a desired fraction.

The policy specified in this case involves the following general characteristics:

1. The range of values for y is 0 to 100.
2. When the fraction observed is equal to the goal, the payoff value, y, should equal 50.
3. As the fraction observed becomes greater than the goal, the value slowly decreases below 50 to a minimum of 0.
4. As the fraction observed becomes less than the goal, the value slowly increases above 50 to a maximum of 100.
5. For any goal fraction, g, the increasing values moving from 50 to 100 will change at the same rate as the decreasing values from 50 to 0.
6. For any goal fraction, g, the complete range of values from 0 to 100 will be possible.

The policy is reflected by the following expressions. The developments of these expressions are given in Appendix C.

When $0 < g \leq .5$ and $0 \leq f \leq 2g$

$$y = 50 + \frac{50}{g^3}(g - f)^3$$

For the special case

$g = 0$ and $f \neq 0$, then $y = 0$
$g = 0$ and $f = 0$, then $y = 50$

When $0 < g \leq .5$ and $2g < f \leq 1$

$$y = 0$$

When $.5 < g < 1$ and $(2g - 1) \leq f \leq 1$

$$y = 50 + \frac{50}{(1 - g)^3}(g - f)^3$$

For the special cases

$g = 1$ and $f \neq 1$, then $y = 100$
$g = 1$ and $f = 1$, then $y = 50$

When $.5 < g < 1$ and $0 \leq f < (2g - 1)$

$$y = 100$$

Notice that a small power (3) was chosen to approximate this policy. Other odd powers could be used if appropriate.

Figure 11 displays the sketches for three values of g.

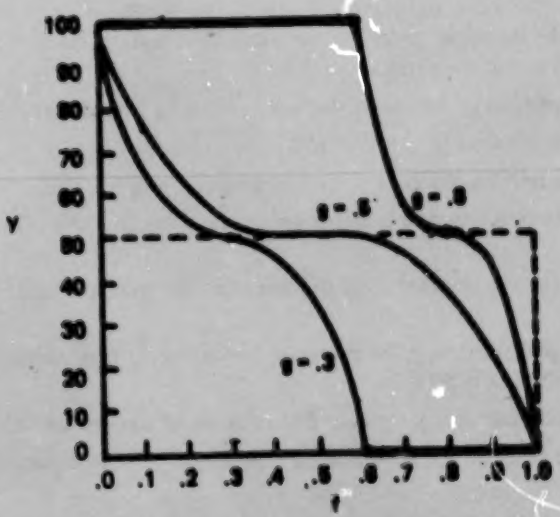


Figure 11. Y (payoff) = function of f(fraction fill) for g = .3, g = .5, g = .8.

Values of y are presented in Table 4 for various values of f and g.

Table 4 Y(Payoff) = Function of g(goal) and f(fraction fill)

		FRACTION FILLED										
		.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
	1.0	100	100	100	100	100	100	100	100	100	100	50
	.9	100	100	100	100	100	100	100	100	100	50	0
	.8	100	100	100	100	100	100	100	56	50	44	0
	.7	100	100	100	100	100	65	62	50	48	35	0
G	.6	100	100	100	71	56	51	50	49	44	29	0
O	.5	100	76	61	53	50	50	50	47	39	24	0
A	.4	100	71	56	51	50	49	44	29	0	0	0
L	.3	100	65	52	50	48	35	0	0	0	0	0
	.2	100	56	50	44	0	0	0	0	0	0	0
	.1	100	50	0	0	0	0	0	0	0	0	0
	0	50	0	0	0	0	0	0	0	0	0	0

In actual operation with this component it may be desirable to use a default option by imposing the condition $y = 50$ when $g = 0$ or $g = 1$. This results in an "on target" assumption for $g = 0$ or $g = 1$ for any values of f. If this approach is used for default, then it would be necessary to use a g value very close to zero to represent a goal of 0 and to use a g value very close to one to represent a goal of 1.

Example 4: Value to the Air Force as a Function of Several Components

This example involves weighting several components to express value in a way that controls the "contribution" or "relative weighting" of each component in the composite.

In this case the policy is described as follows:

1. The range of the composite value, y, is from 0 to 1000.

2. The "relative weighting" or "contribution" of a composite is determined by converting the range of each component into a specified fraction of the total composite indicated as follows.

Letting

- x_i = value of variable i, $i = 1, \ldots, n$
- f_i = fraction of composite to be used by variable i
- h_i = high value of variable i
- l_i = low value of variable i
- h_c = high value of composite
- l_c = low value of composite

Then the model developed in Appendix D to express the previous policy is

$$y = l_c + \sum_{i=1}^{n} f_i \frac{(h_c - l_c)}{(h_i - l_i)} (x_i - l_i)$$

For example, let

$n = 2$

$h_1 = 100$	$h_2 = 50$	$h_c = 1000$
$l_1 = 0$	$l_2 = 0$	$l_c = 0$
$f_1 = .6$	$f_2 = .4$	

$$y = l_c + f_1 \frac{(h_c - l_c)}{(h_1 - l_1)}(x_1 - l_1) + f_2 \frac{(h_c - l_c)}{(h_2 - l_2)}(x_2 - l_2)$$

$$y = 0 + .6 \frac{(1000 - 0)}{(100 - 0)}(x_1 - 0) + .4 \frac{(1000 - 0)}{(50 - 0)}(x_2 - 0)$$

$$y = 6x_1 + 8x_2$$

Figures 12 and 13 present several sketches of this function.

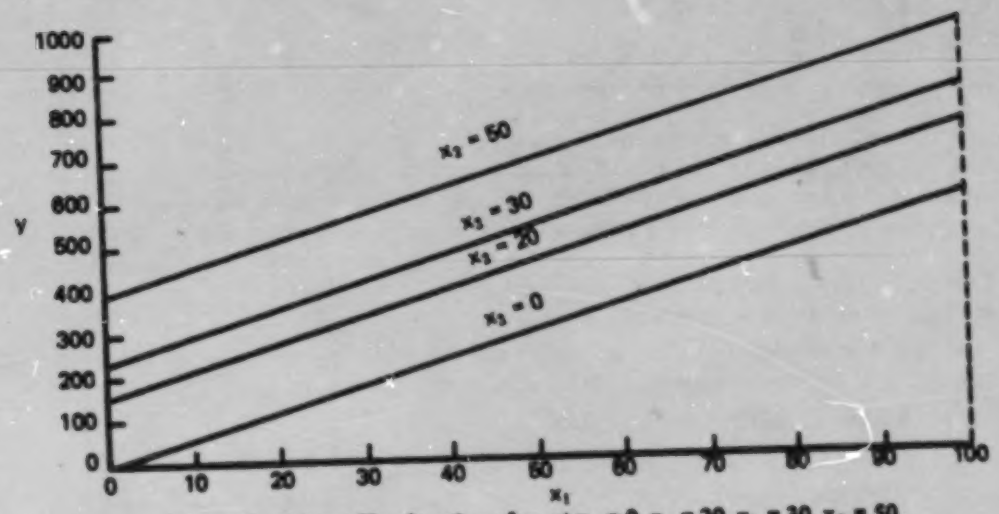


Figure 12. Y (payoff) = function of x_1 at $x_2 = 0, x_2 = 20, x_2 = 30, x_2 = 50$.

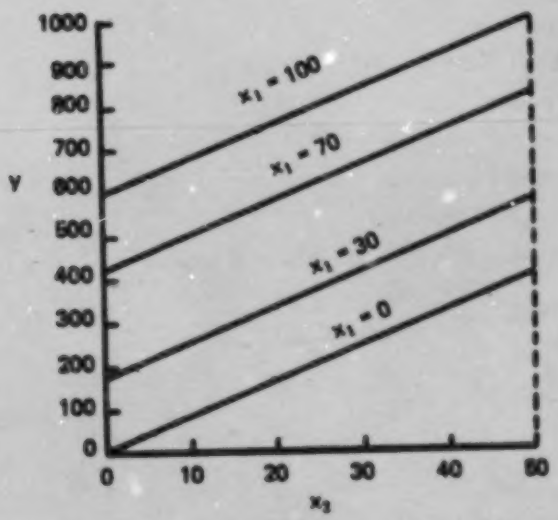


Figure 13. Y (payoff) = function of x_2 at $x_1 = 0, x_1 = 30, x_1 = 70, x_1 = 100$.

Values of y are presented in Table 5 for various combinations of x_1 and x_2.

Table 5. Y (Payoff) = Function of x_1 and x_2 ($y = 6x_1 + 8x_2$)

		x_1										
		0	10	20	30	40	50	60	70	80	90	100
	50	400	460	520	580	640	700	760	820	880	940	1000
	40	320	380	440	500	560	620	680	740	800	860	920
x_2	30	240	300	360	420	480	540	600	660	720	780	840
	20	160	220	280	340	400	460	520	580	640	700	760
	10	80	140	200	260	320	380	440	500	560	620	680
	0	0	60	120	180	240	300	360	420	480	540	600

IV. GENERAL PURPOSE MODELS FOR POLICY-SPECIFYING

It was indicated in Section I that policy-specifying required repeated creation of new models with different properties in an effort to produce output values that are acceptable to the policy maker. Section II (and the corresponding Appendixes A through D) gave specific examples of models with interactions among various powers of variables. If the processes, described in Appendixes A through D, had to be repeated everytime a new model was desired, each policy-specifying cycle would be extremely slow. This repetitious and slow process of imposing restrictions for each different situation provides good practice for the model maker but it slows and impedes (and could possibly destroy) the policy-specifying process. To make policy-specifying a viable approach for representing value judgments, several general forms were developed. By varying parameter settings it is easy to generate many different models and examine the results quickly. This section contains the three general forms that were developed with examples of how they can be used to create specific models. At the end, the three general forms are presented together to provide a pictorial aid to policy-specifying. In this section all policy-specified models are written using FORTRAN expressions. This provides for accurate communication of the models and ease of implementation on a computer.

Definitions

The parameters that will be used to describe the general models are described as follows. Values for these parameters are determined by policy. Referring to the following information.

- A(1) = the smaller control value of variable A
- A(2) = the larger control value of variable A
- D(1) = the smaller control value of variable D
- D(2) = the larger control value of variable D
- Y(1,1) = the Y value corresponding to A(1), D(1)
- Y(2,1) = the Y value corresponding to A(2), D(1)
- Y(1,2) = the Y value corresponding to A(1), D(2)
- Y(2,2) = the Y value corresponding to A(2), D(2)
- AEXP = the polynomial exponent for variable A

DEXP = the polynomial exponent for variable D

KONA = control for variable A characteristics

KOND = control for variable D characteristics. KONA and KOND are used to specify the locations and movements of slopes and inflection points. KONA will take the value of 1 to specify control with reference to A(1) and the value 2 to specify control with reference to A(2). Similarly KOND will take the value of 1 to specify control with reference to D(1) and the value 2 to specify control with reference to D(2).

An example of locations for the A's, D's, and Y's is presented as Figure 14.

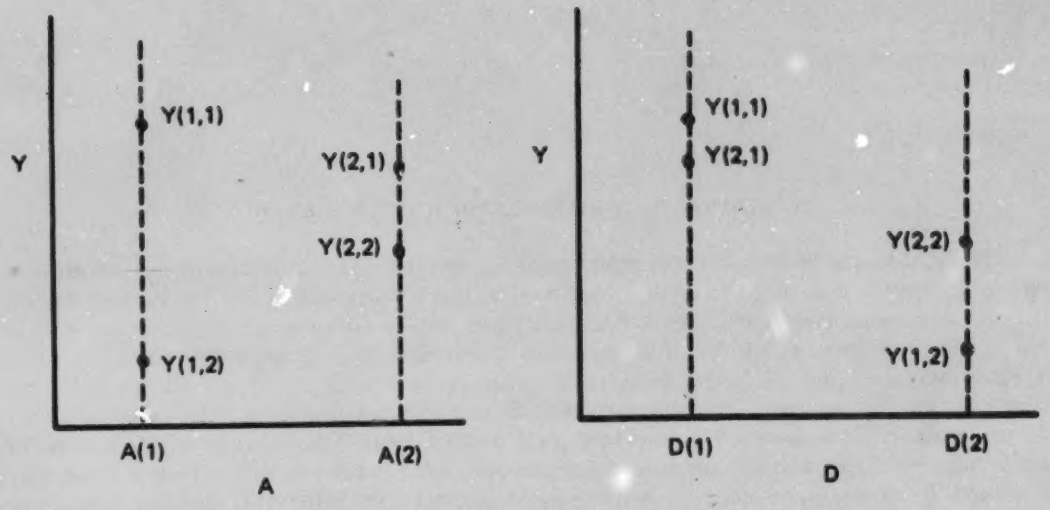


Figure 14. Y (payoff) = function of A and D. An example of locations of parameters.

Model 1

The first general model allows for expressing the composite value Y as a function of any two variables, A and D, using a general polynomial form with easy control of the location of slopes of zero and critical Y values. Appendix E describes the development of this model.

Model 1 is expressed as

$$Y = B(1) + B(2) * (A - A(KONA)) ** AEXP + B(3) * (D - D(KOND)) ** DEXP + B(4) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** DEXP)$$

Where

B(1) = Y (KONA, KOND)

B(2) = (Y(KONACH, KOND) - Y(KONA, KOND))/((A(KONACH) - A(KONA)) ** AEXP)

B(3) = (Y(KONA, KONDCH) - Y(KONA, KOND))/((D(KONDCH) - D(KOND)) ** DEXP)

B(4) = (Y(KONA, KOND) - Y(KONA, KONDCH) - Y(KONACH, KOND) + Y(KONACH, KONDCH))/(((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP))

Also note that * means "multiplication" and ** means "exponentiation."

Where KONACH = 3 - KONA
= 2 when KONA =1
= 1 when KONA = 2
KONDCH = 3 - KOND
= 2 when KOND = 1
= 1 when KOND = 2

This model has the following characteristics under control by KONA and KOND:

1. For every value of D the slope for variable A is zero at A(KONA) and no other values of A.

2. For every value of A the slope for variable D is zero at D(KOND) and no other values of D.

3. For any value of AEXP and for all values of D the variable A and the slope Y with respect to A (called A-slope) is either always increasing or always decreasing between A(1) and A(2). Note that higher values of AEXP result in slower changes in Y as A changes near A(KONA) and faster changes in Y as A changes near A(KONACH).

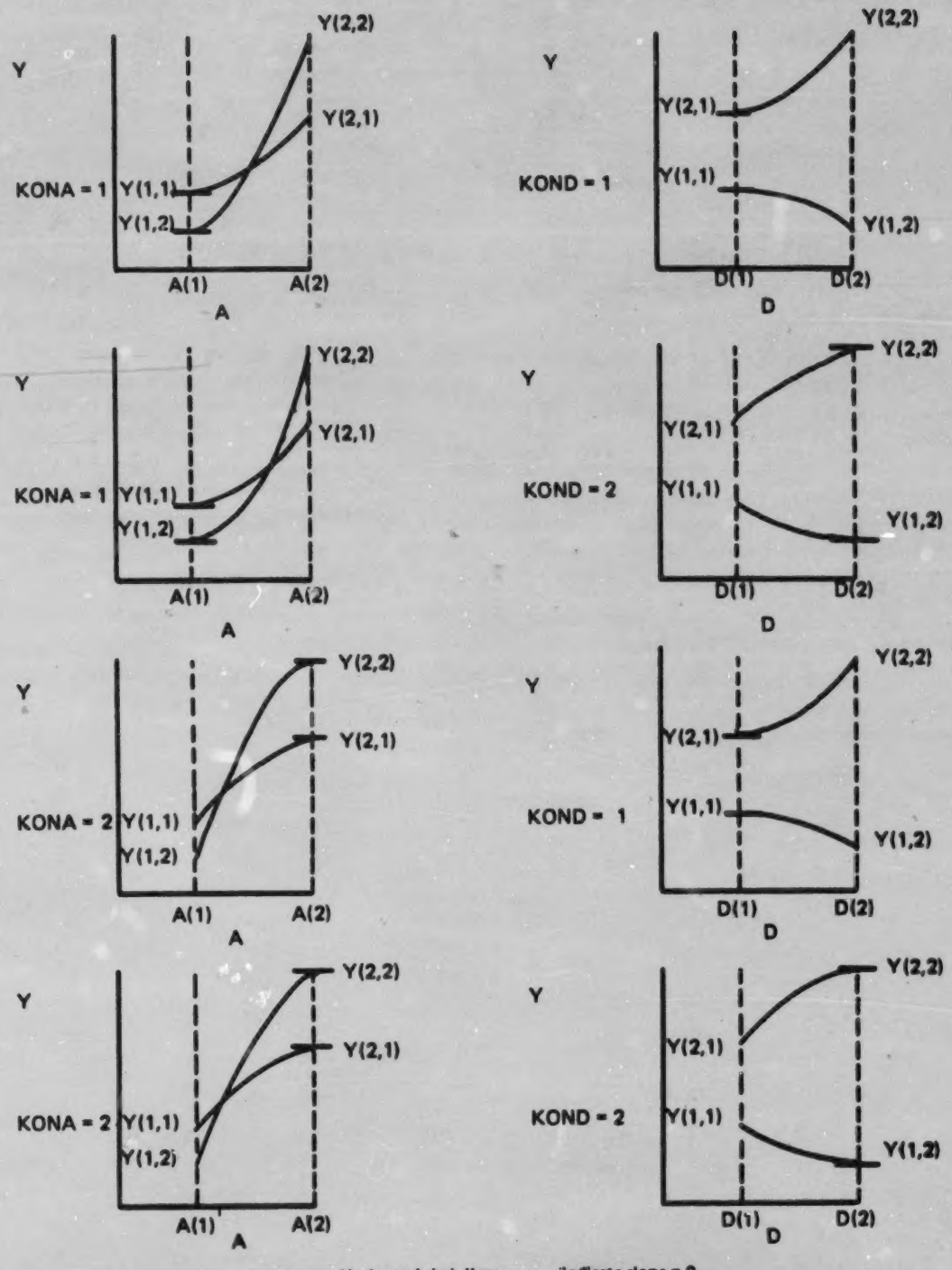
4. For any value of DEXP and for all values of A, the variable D and the slope of Y with respect to D (called D-slope) is either always increasing or always decreasing between D(1) and D(2). Note that higher values of DEXP result in slower changes in Y as D changes near D(KOND) and faster changes in Y as D changes near D(KONDCH).

Figure 15 shows sketches of curves at the critical control values for a possible policy specification.

This general form will now be used to represent Example 2 above where Y is a function of time used, T; fraction of fill, F; and job-importance, K.

In this situation the variable A corresponds to T (days used) and variable D corresponds to F (fraction of fill). As defined before, K is the job importance indicator. Then we can specify the parameters in the general model to obtain the specific model.

A(1)	=	0, the smaller control value of T (days used)
A(2)	=	180, the larger control value of T (days used)
D(1)	=	0, the smaller control value of F (fraction fill)
D(2)	=	1.0, the larger control value of F (fraction fill)
Y(1,1)	=	K, Y value at T = 0, F = 0
Y(2,1)	=	100, Y value at T = 180, F = 0
Y(1,2)	=	0, Y value at T = 0, F = 1.0
Y(2,2)	=	K, Y value at T = 180, F = 1.0
AEXP	=	1, the change in Y is constant as T changes
DEXP	=	1, the change in Y is constant as F changes
KONA = 1 KOND = 1		Setting of KONA and KOND can be either 1 or 2, since there is no requirement of control for slopes.
KONACH =		3 - KONA = 2
KONDCH =		3 - KOND = 2



Note: Horizontal dark lines —— indicate slope = 0

Figure 15. **Y (payoff) = function of A and D. Examples of parameter settings for Model 1.**

Substitution yields

B(1) = Y(KONA, KOND) = Y(1,1) = K

B(2) = (Y(KONACH, KOND) - Y(KONA, KOND))/((A(KONACH) - A(KONA)) ** AEXP)

= (Y(2,1) - Y(1,1)/((A(2) - A(1)) ** 1)

= (100 - K)/(180 - 0) ** 1 = (100 - K)/180

B(3) = (Y(KONA, KONDCH) - Y(KONA, KOND))/((D(KONDCH) - D(KOND)) ** DEXP)

= (Y(1,2) - Y(1,1))/((D(2) - D(1)) ** 1)

= (0 - K)/(1.0 - 0) ** 1 = –K

B(4) = (Y(KONA, KOND) - Y(KONA, KONDCH) - Y(KONACH, KOND) + Y(KONACH, KONDCH))/(((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP))

= (Y(1,1) - Y(1,2) - Y(2,1) + Y(2, 2))/(((A(2) - A(1)) ** 1) * ((D(2) - D(1)) ** 1))

= (K - 0 - 100 + K)/(((180 - 0) ** 1) * ((1.0 - 0) ** 1))

= (2 * K - 100)

and the final model written similar to the original form of Example 2 on page 16 is

$$Y = K + \frac{(100 - K)}{180}T + (-K)F + \frac{(2K - 100)}{180}(TF)$$

Recalling that Y(1,1) = K and Y(2,2) = K, to generate this function when K = 25, requires Y(1,1) = 25 and Y(2,2) = 25. The output of this general model and its parameter settings are shown in Table 6. The results are identical to Table 2. The following parameters are used to control the range of A and D used in generating Y values from the function.

KSTARA = the first value of A

KSTOPA = the last value of A

KINCA = the amount that A is incremented

KSTARD = the first value of D

KSTOPD = the last value of D

KINCD = the amount that D is incremented

The parameter settings and output of this function for K = 75 is shown as Table 7. This output is the same as Table 3.

Now assume that the policy maker decides that everything is fine except that instead of constant changes in Y values the change in Y values should be gradual near T = 0 and F = 0 (corresponding to A(1) and D(1)). This requies control of slopes and is accomplished by introducing a polynomial of degree 2 or more and by requiring the slopes of Y with respect to A (A-slope) for all D to be zero at A(1) and the slope of Y with respect to D (D-slope) for all A to be zero at D(1).

Assume then that

AEXP = 2

DEXP = 2

KONA = 1 (makes slopes = 0 at A(1) = 0)

KOND = 1 (makes slopes = 0 at D(1) = 0)

The output of this new function for K = 25 and K = 75 is shown as Tables 8 and 9.

Table 6. **Y (Payoff) = Function of t (time used) and f (fraction fill) for k = 25**

```
MODEL=1

Y(1,1)=  25 Y(2,1)= 100 Y(1,2)=   0 Y(2,2)=  25
AEXP= 1 DEXP= 1 KONA= 1 KOND= 1
A(1)=  0 A(2)=180 D(1)=  0 D(2)=100
KSTARA=180 KSTOPA=  0 KINCA=-15
KSTARD=  0 KSTOPD=100 KINCD= 10


Y =   25 + [< .4167+00>*(T -   0)** 1] + [<-25.00 +00>*(F -   0)** 1]
         + [<-.2778-00>*[(T -   0)** 1]*[(F -   0)** 1]]
```

	FILL										
	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	93	85	78	70	63	55	48	40	33	25
165	94	87	80	73	65	58	51	44	37	30	23
150	87	81	74	68	61	54	48	41	34	28	21
135	81	75	69	63	56	50	44	37	31	25	19
120	75	69	63	58	52	46	40	34	28	22	17
105	69	63	58	52	47	42	36	31	25	20	15
90	62	58	53	48	43	38	33	28	23	18	13
75	56	52	47	43	38	33	29	24	20	15	10
60	50	46	42	38	33	29	25	21	17	12	8
45	44	40	36	33	29	25	21	17	14	10	6
30	37	34	31	27	24	21	17	14	11	7	4
15	31	28	25	23	20	17	14	11	8	5	2
0	25	23	20	18	15	13	10	8	5	3	0

TIME

Table 7. Y (Payoff) = Function of t (time used) and f (fraction fill) for k = 75

```
MODEL=1

Y(1,1)=  75 Y(2,1)= 100 Y(1,2)=   0 Y(2,2)=  75
AEXP= 1 DEXP= 1 KONA= 1 KOND= 1
A(1)=  0 A(2)=180 D(1)=  0 D(2)=100
KSTARA=180 KSTOPA=  0 KINCA=-15
KSTARD=  0 KSTOPD=100 KINCD= 10

Y =   75 + [< .1389+00>*(T -   0)** 1] + [<-75.00 +00>*(F -   0)** 1]
         + [< .2778-00>*(T -   0)** 1]*[(F -   0)** 1]
```

TIME \ FILL	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	97	95	92	90	87	85	82	80	77	75
165	98	95	92	89	86	83	80	77	75	72	69
150	96	92	89	86	82	79	76	72	69	66	62
135	94	90	86	82	79	75	71	67	64	60	56
120	92	87	83	79	75	71	67	62	58	54	50
105	90	85	80	76	71	67	62	57	53	48	44
90	87	82	77	72	67	62	57	52	47	42	37
75	85	80	75	69	64	58	53	47	42	37	31
60	83	77	72	66	60	54	48	42	37	31	25
45	81	75	69	62	56	50	44	37	31	25	19
30	79	72	66	59	52	46	39	32	26	19	12
15	77	70	63	56	49	42	35	27	20	13	6
0	75	68	60	53	45	38	30	23	15	8	0

Table 8. Y (Payoff) = Second Degree Function of t (time used) and Second Degree Function of f (fraction fill) for k = 25

```
MODEL=1

Y(1,1)=  25 Y(2,1)= 100 Y(1,2)=   0 Y(2,2)=  25
AEXP= 2 DEXP= 2 KONA= 1 KOND= 1
A(1)=  0 A(2)=180 D(1)=  0 D(2)=100
KSTARA=180 KSTOPA=  0 KINCA=-15
KSTARD=  0 KSTOPD=100 KINCD= 10

Y =    25 + [< .2315-02>*(T -   0)** 2] + [<- 25.00-00>*(F -   0)** 2]
          + [<-.1543-02,>*[( T -   0)** 2]*[( F -   0)** 2]]

                                F I L L

          .0  .10  .20  .30  .40  .50  .60  .70  .80  .90 1.00

    180 100   99   97   93   88   81   73   63   52   39   25
    165  88   87   85   82   77   71   64   55   45   34   21
    150  77   76   75   72   68   62   56   48   39   29   17
    135  67   67   65   62   59   54   48   41   33   24   14
    120  58   58   56   54   51   47   41   35   28   20   11
T   105  51   50   49   47   44   40   35   30   24   16    9
I    90  44   43   42   40   38   34   30   25   20   13    6
M    75  38   38   37   35   33   30   26   22   16   11    4
E    60  33   33   32   31   28   26   22   18   14    9    3
     45  30   29   29   27   25   23   20   16   12    7    2
     30  27   27   26   25   23   20   18   14   10    6    1
     15  26   25   25   23   21   19   16   13    9    5    0
      0  25   25   24   23   21   19   16   13    9    5    0
```

Table 9. **Y (Payoff) = Second Degree Function of t (time used) and Second Degree Function of f (fraction fill) for k = 75**

```
MODEL=1

Y(1,1)=  75 Y(2,1)= 100 Y(1,2)=   0 Y(2,2)=  75
AEXP= 2 DEXP= 2 KONA= 1 KOND= 1
A(1)=  0 A(2)=180 D(1)=  0 D(2)=100
KSTARA=180 KSTOPA=  0 KINCA=-15
KSTARD=  0 KSTOPD=100 KINCD= 10

Y =   75 + [< .7716-03>*(T -   0)** 2] + [<= 75.00-00>*(F -   0)** 2]
         + [< .1543-02>*[(T -   0)** 2]*[(F -   0)** 2]]
```

		FILL									
	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	100	99	98	96	94	91	88	84	80	75
165	96	96	95	93	91	88	84	80	75	69	63
150	92	92	91	89	86	82	78	73	67	60	52
135	89	89	87	85	82	77	72	66	59	51	42
120	86	86	84	81	78	73	67	60	52	43	33
T 105	84	83	81	78	74	69	63	55	46	37	26
I 90	81	81	79	76	71	66	59	51	41	31	19
M 75	79	79	77	73	69	63	55	47	37	26	13
E 60	78	77	75	72	67	60	53	44	33	22	8
45	77	76	74	70	65	59	51	41	31	18	5
30	76	75	73	69	64	57	49	40	29	16	2
15	75	74	72	68	63	57	48	39	27	15	1
0	75	74	72	68	63	56	48	38	27	14	0

If the policy maker decides that the changes near T = 0 and F = 0 should be more gradual, then higher exponents could be used. Also the changes might be more gradual for D than for A.

Assume that

AEXP = 3

DEXP = 5

The outputs of this new function for K = 25 and K = 75 are shown as Tables 10 and 11. Three-dimensional representations of these two models are shown as Figures 16 and 17.

Table 10. Y (Payoff) = Third Degree Function of t (time used) and Fifth Degree Function of f (fraction fill) for k = 25

```
MODEL=1

Y(1,1)=  25 Y(2,1)= 100 Y(1,2)=   0 Y(2,2)=  25
AEXP= 3 DEXP= 5 KONA= 1 KOND= 1
A(1)=  0 A(2)=180 D(1)=  0 D(2)=100
KSTARA=180 KSTOPA=  0 KINCA=-15
KSTARD=  0 KSTOPD=100 KINCD= 10

Y =   25 + [< .1286-04>*(T -   0)** 3] + [<-25.00-00>*(F -   0)** 5]
         + [<-.8573-05>*[(T -   0)** 3]*[(F -   0)** 5]]
```

TIME \ FILL	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	100	100	100	99	98	94	87	75	56	25
165	83	83	83	83	82	81	78	72	62	45	19
150	68	68	68	68	68	67	64	59	51	37	14
135	57	57	57	57	56	55	53	49	42	29	11
120	47	47	47	47	47	46	44	41	34	24	7
105	40	40	40	40	40	39	37	34	28	19	5
90	34	34	34	34	34	33	32	29	24	16	3
75	30	30	30	30	30	30	28	26	21	14	2
60	28	28	28	28	28	27	26	23	19	12	1
45	26	26	26	26	26	25	24	22	18	11	0
30	25	25	25	25	25	25	23	21	17	10	0
15	25	25	25	25	25	25	24	23	21	17	10	0
0	25	25	25	25	25	25	24	23	21	17	10	0

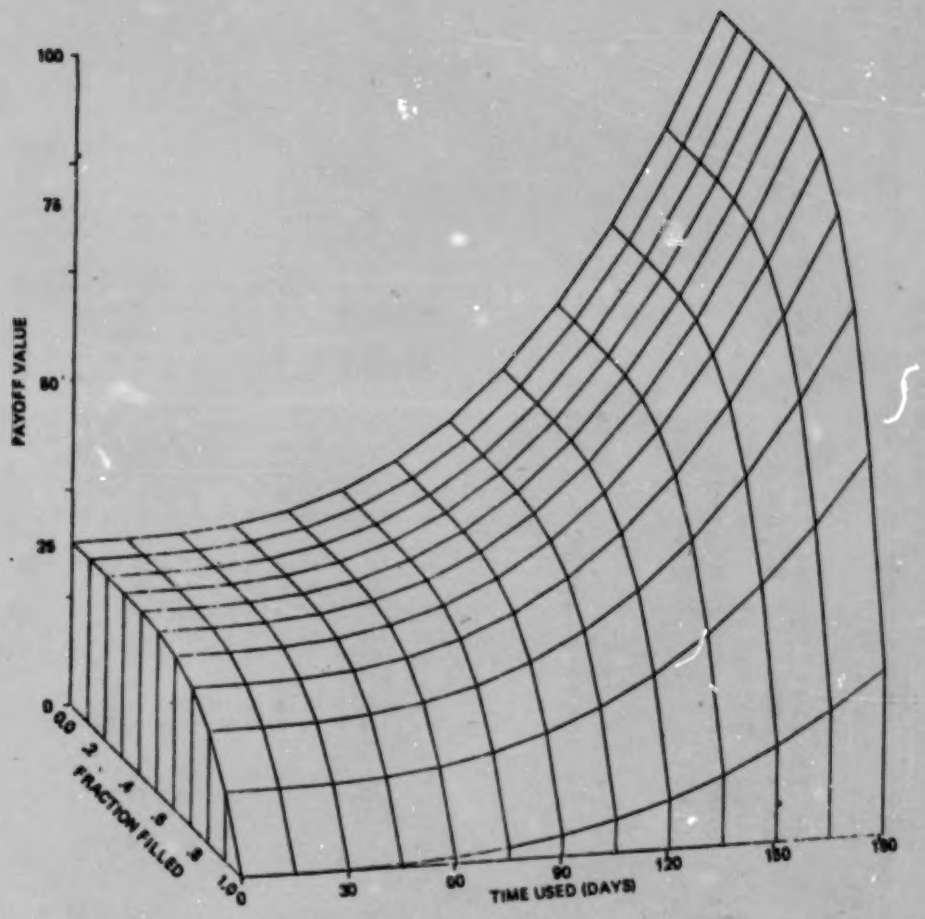


Figure 16. Y = third degree function of t(time used) and fifth degree function of f(fractional fill) for k = 25.

Table 11. **Y (Payoff) = Third Degree Function of t (time used) and Fifth Degree Function of f (fraction fill) for k = 75**

```
MODEL=1

Y(1,1)=  75 Y(2,1)= 100 Y(1,2)=   0 Y(2,2)=  75
AEXP= 3 DEXP= 5 KONA= 1 KOND= 1
A(1)=  0 A(2)=180 D(1)=  0 D(2)=100
KSTARA=180 KSTOPA=  0 KINCA=-15
KSTARD=  0 KSTOPD=100 KINCD= 10

Y =   75 + [< .4287-05>*(T -   0)** 3] + [<-.75.00-00>*(F -   0)** 5]
         + [< .8573-05>*[(T -   0)** 3]*[(F -   0)** 5]]
```

TIME \ FILL	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	100	100	100	100	99	98	96	92	85	75
165	94	94	94	94	94	93	91	88	82	73	58
150	89	89	89	89	89	88	86	82	74	62	43
135	86	86	86	85	85	84	81	76	68	54	32
120	82	82	82	82	82	81	78	72	63	47	22
105	80	80	80	80	79	78	75	69	59	42	15
90	78	78	78	78	77	76	73	67	56	38	9
75	77	77	77	77	76	75	71	65	53	35	5
60	76	76	76	76	75	74	70	64	52	33	3
45	75	75	75	75	75	73	70	63	51	32	1
30	75	75	75	75	74	73	69	63	51	31	0
15	75	75	75	75	74	73	69	62	50	31	0
0	75	75	75	75	74	73	69	62	50	31	0

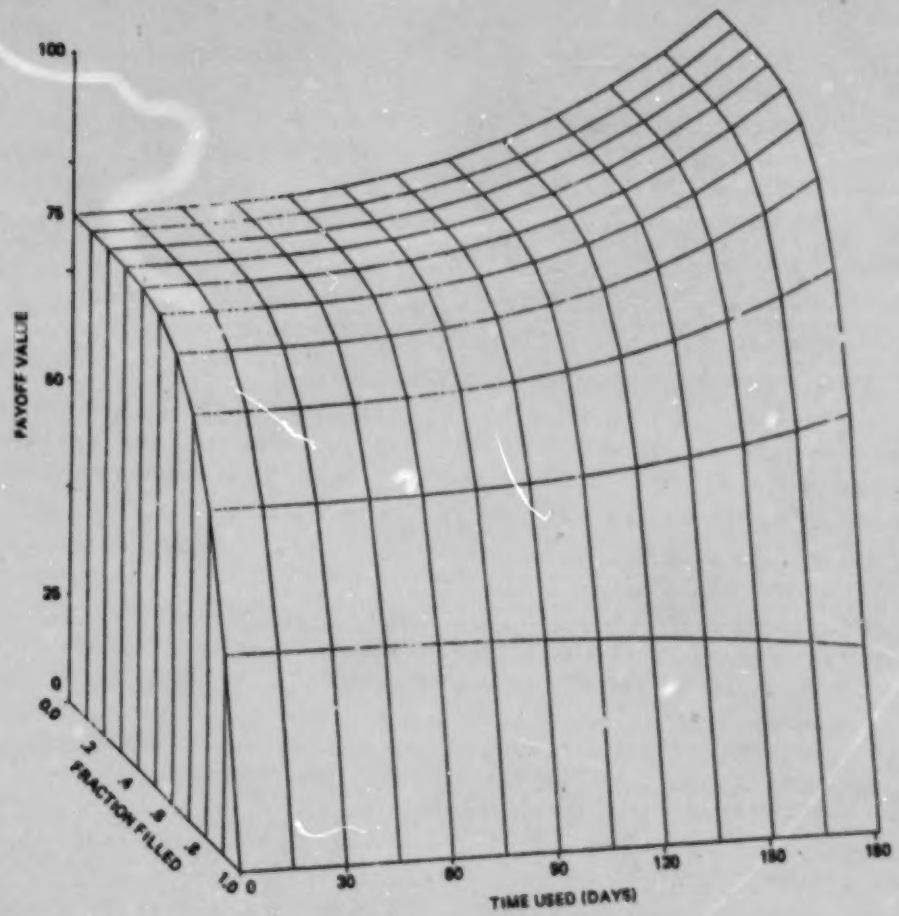


Figure 17. Y = third degree function of f(time used) and fifth degree function of f(fractional fill) for k = 75.

Model 2

The second general model allows for expressing the composite value Y as a function of any two variables A and D using a general polynomial form as in Model 1. However, this model allows for more complex control of the slopes so that policy expressions that reflect statements about maximum or minimum values can be easily specified. Model 2 is developed in Appendix F.

Model 2 is expressed as

```
Y   =   B(1) + B(2) * (A - A(KONA)) ** AEXP
            + B(3) * (D - D(KOND)) ** (DEXP - 1)
        + B(4) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** DEXP)
        + B(5) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** (DEXP - 1))
        + B(6) * (D - D(KOND)) ** DEXP
```

Where

```
B(1) =   Y(KONA, KOND)
B(2) =   (Y(KONACH, KOND) - Y(KONA, KOND))/((A(KONACH) - A(KONA)) ** AEXP)
```

B(3) = (DEXP * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((D(KONDCH) - D(KOND)) ** (DEXP -1))

B(4) = (Y(KONACH, KONDCH) - Y(KONACH, KOND) + ((DEXP -1)* (Y(KONA, KONDCH) - Y(KONA, KOND))))/(((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP))

B(5) = (-B(3))/(A(KONACH) - A(KONA)) ** AEXP

B(6) = ((-B(3)) * (DEXP - 1))/(DEXP * (D(KONDCH) - D(KOND)))

This model has the following characteristics, the first three being the same as for Model 1:

1. For every value of D the A-slope is zero at A(KONA) and no other values of A.

2. For every value of A the D-slope is zero at D(KOND).

3. For any value of AEXP and for all values of D the variable A and A-slope are either always increasing or always decreasing between A(1) and A(2). Note as before that higher values of AEXP result in slower changes in Y as A changes near A(KONA) and faster changes in Y as A changes near A(KONACH).

4. For any value of DEXP $\geq$ 3, the D-slope variable is zero at D(KONDCH) and A(KONA).

5. For any value of DEXP $\geq$ 3, and for A(KONA) there is only one inflection point for D-slope between the zero slope at D(KOND) and the zero slope at D(KONDCH).

6. For any value of DEXP $\geq$ 3 and for A(KONACH) the variable D and the D-Slope are either always increasing or always decreasing between D(1) and D(2). Note that higher values of DEXP result in slower changes in Y as D changes near D(KOND) and faster changes near D(KONDCH).

7. The previous restrictions result in the movement of D-slope = 0 from A(KONA), D(KONDCH) to A(KONACH), D(KOND). This allows for policy specification requiring ridges or valleys indicated by dotted lines on the right side of Figure 18. These sketches indicate how Model 2 might appear with selected settings of Y's and D's for all combinations of KONA and KOND.

The general form of Model 2 will now be used to represent Example 1 above where Y is a function of Aptitude, A, and Job Difficulty, D.

Here let variable A correspond to A (aptitude), and variable D correspond to D (difficulty). Now use Model 2 to create the required policy specification.

A(1)	=	40, the smaller control value for A (aptitude)
A(2)	=	95, the larger control value for A (aptitude)
D(1)	=	40, the smaller control value for D (difficulty)
D(2)	=	100, the larger control value for D (difficulty)
Y(1,1)	=	15, Y value at A = 40, D = 40
Y(2,1)	=	35, Y value at A = 95, D = 40
Y(1,2)	=	−250, Y value at A = 40, D = 100, and determined by experience with different values to obtain policy specification 3 for the example.
Y(2,2)	=	100, Y value at A = 95, D = 100
AEXP	=	1, change in Y is constant as A changes
DEXP	=	3, value of DEXP that gives control of maximum values at desired places. Higher values can be tried to observe Y value characteristics.
KONA	=	2, provides for formation of a ridge of maximum Y values from A = 40, D = 40 to A = 95, D = 100

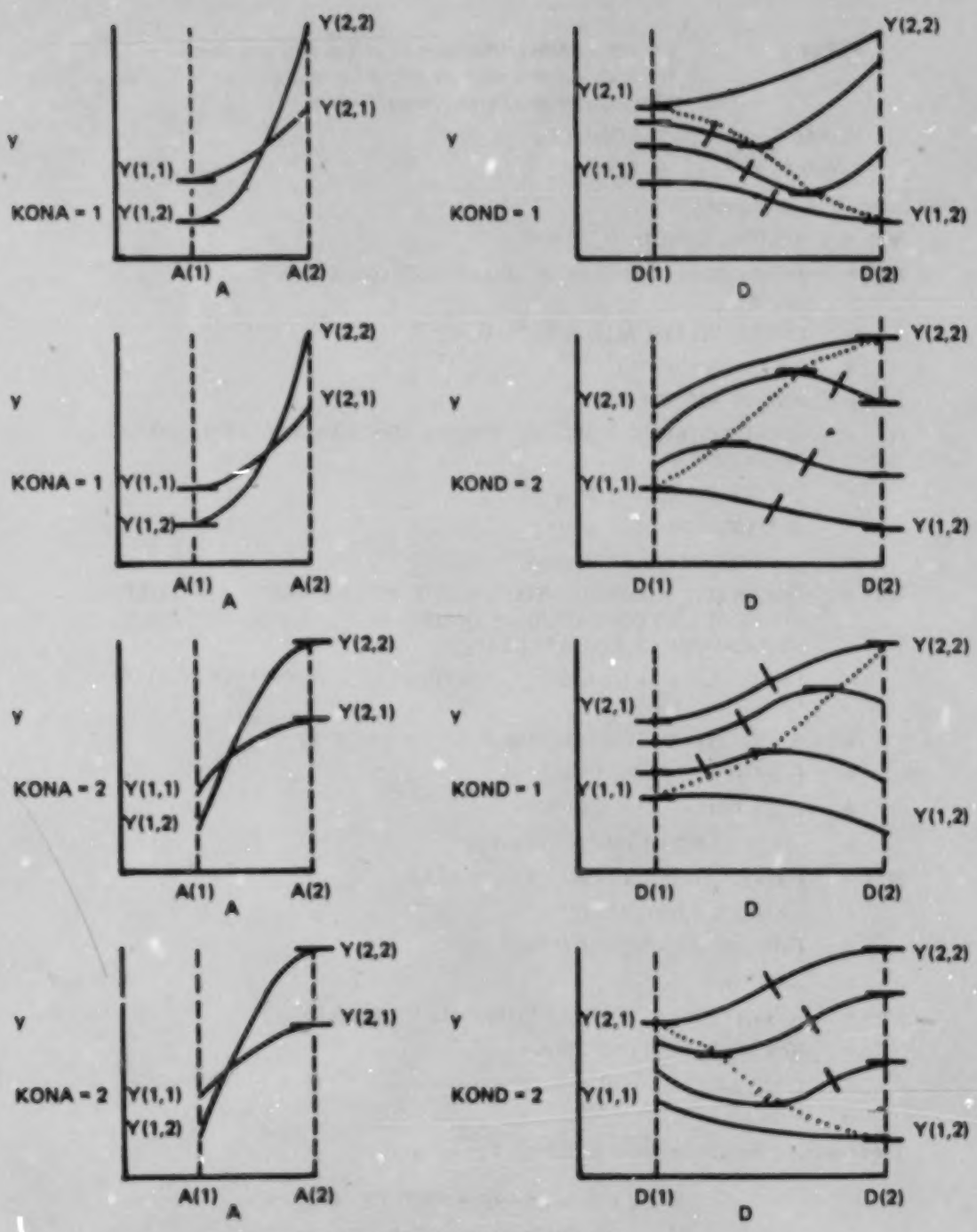


Figure 18. Y (payoff) = function of A and D. Examples of parameter settings for Model 2.

KOND = 1 which provides gradual increase of Y as difficulty changes near D = 40 for all values of aptitude, allows for rapid decrease of Y values as difficulty changes after maximum Y values

KONACH = 3 - KONA = 1

DONDCH = 3 - KOND = 2

Substitution in Model 2 gives

B(1) = Y(KONA, KOND) = Y(2,1) = 35

B(2) = (Y(KONACH), KOND) - Y(KONA, KOND))/((A(KONACH) - A(KONA)) ** AEXP)

= (Y(1,1) - Y(2,1))/((A(1) - A(2)) ** AEXP)

= (15 - 35)/(40-95) ** 1

= −20/−55 = 20/55 = .3636

B(3) = (DEXP * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((D(KONDCH) - D(KOND)) ** (DEXP - 1))

= (3 * (Y(2,2) - Y(2,1)))/(D(2) - D(1)) ** 2

= (3 * (100 - 35))/(100 - 40) ** 2

= (3 * 65)/3600 = 65/1200 = .05417

B(4) = (Y(KONACH, KONDCH) - Y(KONACH, KOND) + ((DEXP - 1) * (Y(KONA, KONDCH) - Y(KONA, KOND))))/ (((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP))

= (Y(1,2) - Y(1,1)) + ((2 * (Y(2,2) - Y(2,1))))/(((A(1) - A(2)) ** 1) * ((D(2) - D(1)) ** 3))

= ((−250 - 15) + 2 * (100 - 35))/((40 - 95) * (100 - 40) ** 3)

= (−265 + 2 * 65)/(−55) * (60) ** 3)

= (−265 + 130)/((−55 * 216 * 1000)

= −135/(−55 * 216 * 1000) = .00001136

B(5) = (−B(3))/(A(KONACH) - A(KONA)) ** AEXP

= (−65/1200)/(A(1) - A(2)) ** 1

= (−65/1200)/(40 - 95) = 65/(1200 * 55)

= .0009848

B(6) = ((-B(3)) * (DEXP - 1))/(DEXP * (D(KONDCH) - D(KOND)))

= ((−65/1200) * 2)/(3 * (100 - 40)

= (−130/1200)/(3 * (60))

= −.0006019

The equation is the same as shown in Example 1 given earlier.

$$Y = 35 + .3636\,(A-95) + .05417\,(D-40)^2 + .00001136\,(A-95)(D-40)^2 + .0009848\,(A-95)(D-40)^2 - .0006019\,(D-40)^3$$

Figure 19 shows the relation between Y and Aptitude at selected Difficulties as well as the relation between Y and Difficulties at selected Aptitudes. These sketches are the same as Figures 4 and 5.

Table 12 contains selected values generated from Model 2 with the parameter specified above. This output is the same as Table 1. Figure 20 is a three-dimensional representation of the model. It is identical to Figure 6.

Model 3

A third general model that follows easily from Model 2 is essentially the same as Model 2, but allows control of inflection points rather than slopes = 0. The development of Generalized Model 3 is described in Appendix G.

Model 3 is expressed exactly as Model 2, except with the following differences for B(3), B(4), and B(6).

B(3) = ((DEXP/2) * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((D(KONDCH) - D(KOND)) ** (DEXP - 1))

B(4) = (Y(KONACH, KONDCH) - Y(KONACH, KOND) + (((DEXP - 2)/2) * (Y(KONA, KONDCH) - Y(KONA, KOND))))/(((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP)

B(6) = ((-B(3)) * (DEXP - 2))/(DEXP * (D(KONDCH) - D(KOND)))

This model has the same properties as Model 2 except that instead of controlling D-slope = 0 this model moves the inflection point of D-slope from A(KONA), D(KONDCH) to A(KONACH), D(KOND). This allows for a slightly different expression of policy.

Figure 21 shows sketches of Model 3. Comparison of Figure 21 with Figure 18 will contrast Models 2 and 3.

The general form of Model 3 will be used for Example 1 with the same parameter settings as used to specify Model 2. This provides an easy comparison between Models 2 and 3. The output of Model 3 is shown in Table 13. Figure 22 is a three-dimensional representation of Model 3, and it can be compared with Figure 20.

Combining the Models

The three models have been combined into an interactive FORTRAN computer program for ease of use. Model 2 and 3 were combined, as described in Appendix H, prior to computer implementation. Appendix I contains the computer program along with an example of its execution.

As an aid to specifying policies, sketches of Models 1, 2, and 3 for all combinations of KONA and KOND can be placed together as shown in Figure 23. a policy maker might be able to better express his policy by selecting a sketch from among the available possibilities. This approach was used during discussions of simulations of the choices made by applicants who are presented an ordered list of jobs.

The goal was to create a model to express "tendency to choose a job," Y, as a function of Rank Order on the list, and a Preference Rating for the job. The Rank Order is to be associated with variable A and the Preference Rating is associated with D. Rank Order ranges from 1 (high) to 15 (low) and Preference Ranges from 9 (high) to 1 (low).

1. The highest value, Y = 100, occurs at Rank = 1 (high) and Preference Rating = 9 (high); i.e., at A(1), D(2). This translates to parameter settings

Y(1,2) = 100 (Y value at A(1), D(2))

2. The lowest value, Y = 1 occurs at Rank = 15 (low) and Preference Rating = 1 (low); i.e., at A(2), D(1). This translates into

Y(2,1) = 1

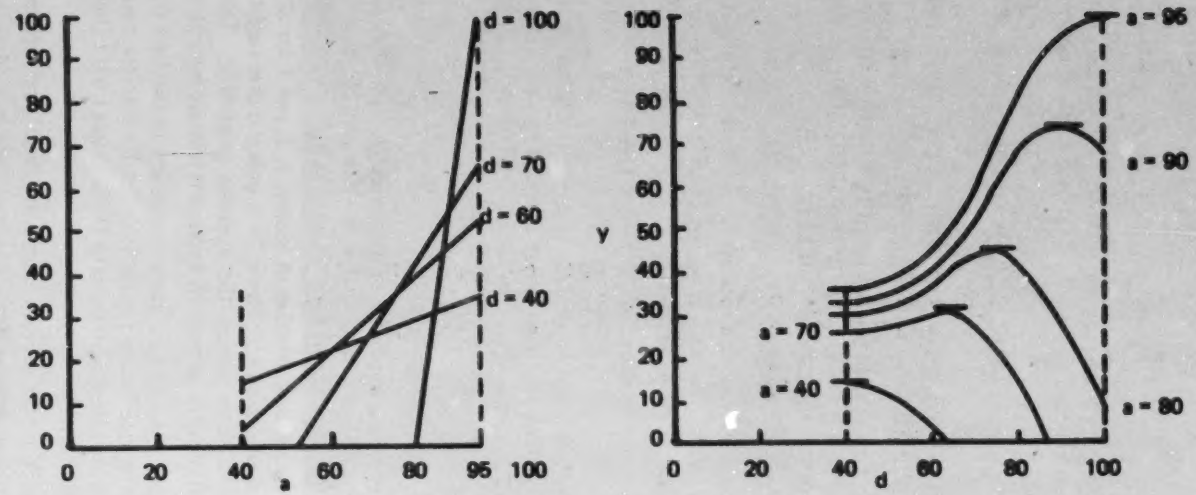


Figure 19. **Y (payoff) = function of a at d = 40, d = 60, d = 70, d = 100.**
Y (payoff) = function of d at a = 40, a = 70, a = 80, a = 90, a = 95.

Table 12. **Y (Payoff) = Function of a (aptitude) and Third Degree Function of d (difficulty) Using Model 2**

```
MODEL=2

Y(1,1)=  15 Y(2,1)=  35 Y(1,2)=-250 Y(2,2)= 100
AEXP= 1 DEXP= 3 KONA= 2 KOND= 1
A(1)= 40 A(2)= 95 D(1)= 40 D(2)=100
KSTARA= 95 KSTOPA= 40 KINCA= -5
KSTARD= 40 KSTOPD=100 KINCD=  5


Y =   35 + [< .3636+00>*(A -  95)** 1] + [< .5417-01>*(D -  40)** 2]
         + [< .1136-04>*[(A -  95)** 1]*[(D -  40)** 3]]
         + [< .9848-03>*[(A -  95)** 1]*[(D -  40)** 2]]
         + [<-.6019-03>*(D -  40)** 3]


                                  D   I   F

        40  45  50  55  60  65  70  75  80  85  90  95 100

    95  35  36  40  45  52  59  67  76  83  90  95  99 100
    90  33  34  37  42  48  54  60  65  70  73  74  73  68
    85  31  32  35  39  43  48  52  55  56  56  53  46  36
    80  30  30  33  36  39  42  44  45  43  39  31  20   5
    75  28  28  30  33  35  36  36  34  30  22  10  -6 -27
A   70  26  27  28  30  31  31  29  24  16   5 -11 -32 -59
P   65  24  25  26  26  26  25  21  14   3 -12 -32 -58 -91
T   60  22  23  23  23  22  19  13   4 -10 -29 -53 -84-123
    55  20  21  21  20  18  13   5  -7 -24 -46 -75-111-155
    50  19  19  19  17  14   7  -3 -17 -37 -63 -96-137-186
    45  17  17  16  14   9   2 -10 -27 -50 -80-117-163-218
    40  15  15  14  11   5  -4 -18 -38 -64 -97-138-189-250
```

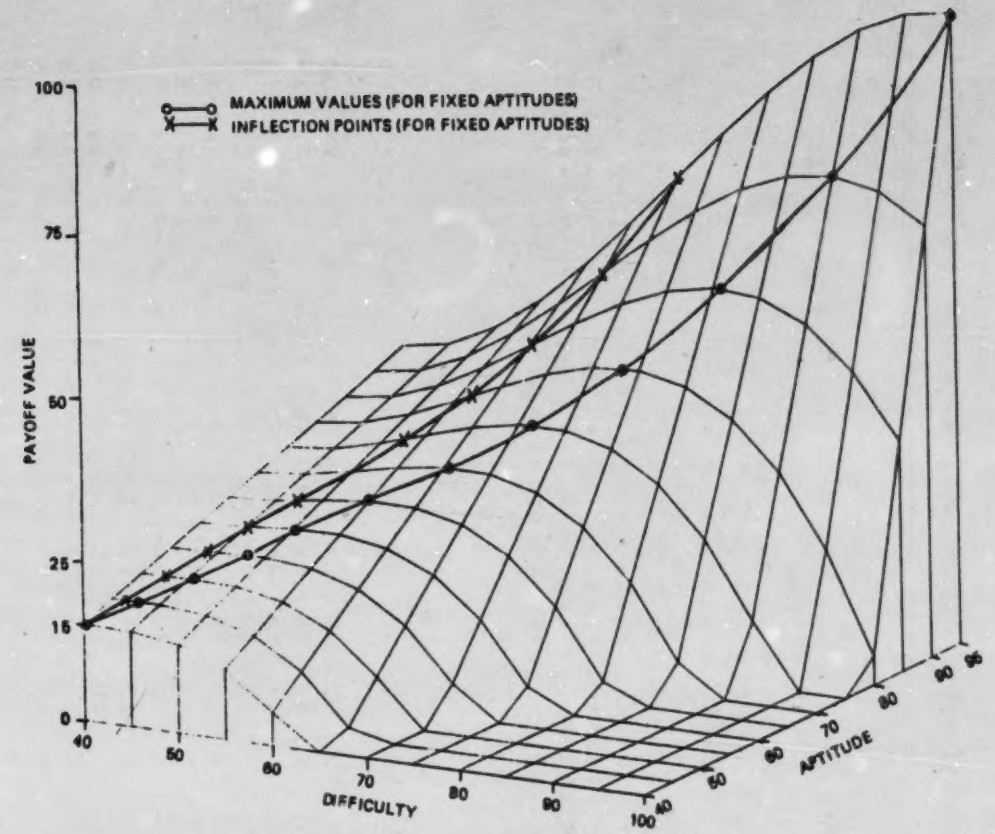
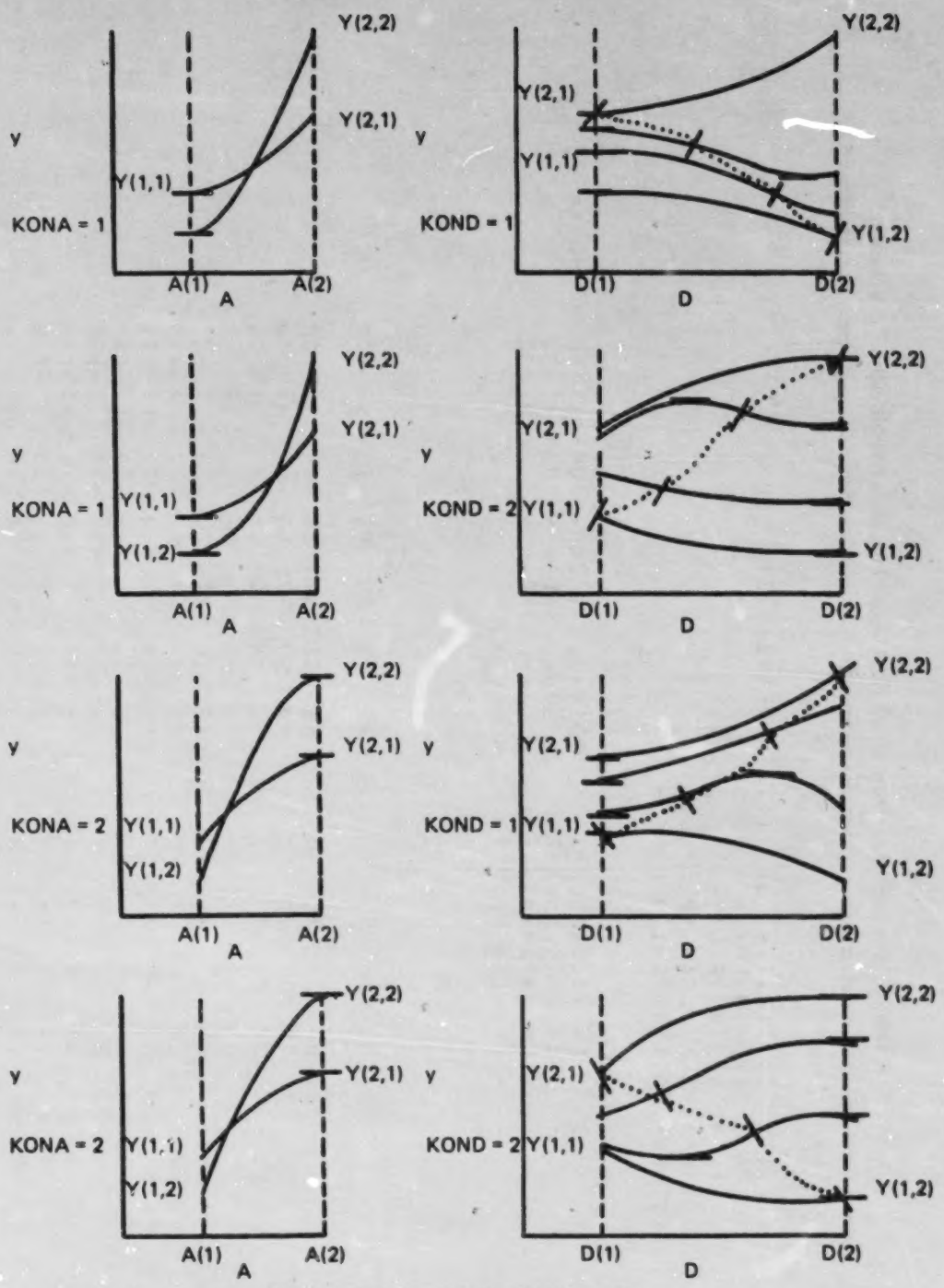


Figure 20. **Three-dimensional view of Model 2. Aptitude-Difficulty Component.**



Note: Horizontal Dark lines (——) indicate slope = 0.
Slanting Dark Lines (/) indicate inflection points.
Dotted Lines (. . . .) indicate connected inflection points.

Figure 21. **Y (payoff) = function of A and D. Examples of parameter settings for Model 3.**

Table 13. **Y (Payoff) = Function of a (aptitude) and Third Degree Function of d (difficulty) Using Model 3.**

```
MODEL=3

Y(1,1)=  15 Y(2,1)=  35 Y(1,2)=-250 Y(2,2)= 100
AEXP= 1 DEXP= 3 KONA= 2 KOND= 1
A(1)= 40 A(2)= 95 D(1)= 40 D(2)=100
KSTARA= 95 KSTOPA= 40 KINCA= -5
KSTARD= 40 KSTOPD=100 KINCD=  5


Y =   35 + [< .3636+00>*(A -  95)** 1] + [< .2708-01>*(D -  40)** 2]
         + [< .1967-04>*[(A -  95)** 1]*[(D -  40)** 3]]
         + [< .4924-03>*[(A -  95)** 1]*[(D -  40)** 2]]
         + [<-.1505-03>*(D -  40)** 3]


                                  D    I    F

        40   45   50   55   60   65   70   75   80   85   90   95  100

   95   35   36   38   41   45   50   55   62   69   76   84   92  100
   90   33   34   35   38   41   45   49   53   57   60   64   66   68
   85   31   32   33   35   37   40   42   44   45   45   43   41   36
   80   30   30   31   32   34   35   35   35   33   29   23   15    5
   75   28   28   29   30   30   30   29   26   21   13    3  -10  -27
A  70   26   26   27   27   27   25   22   17    9   -2  -17  -36  -59
P  65   24   24   25   24   23   20   15    8   -3  -18  -37  -61  -91
T  60   22   22   22   22   20   15    9   -1  -15  -34  -58  -87 -123
   55   20   21   20   19   16   10    2  -11  -27  -50  -78 -112 -155
   50   19   19   18   16   12    6   -5  -20  -39  -65  -98 -138 -186
   45   17   17   16   14    9    1  -11  -29  -51  -81 -118 -164 -218
   40   15   15   14   11    5   -4  -18  -38  -64  -97 -138 -189 -250
```

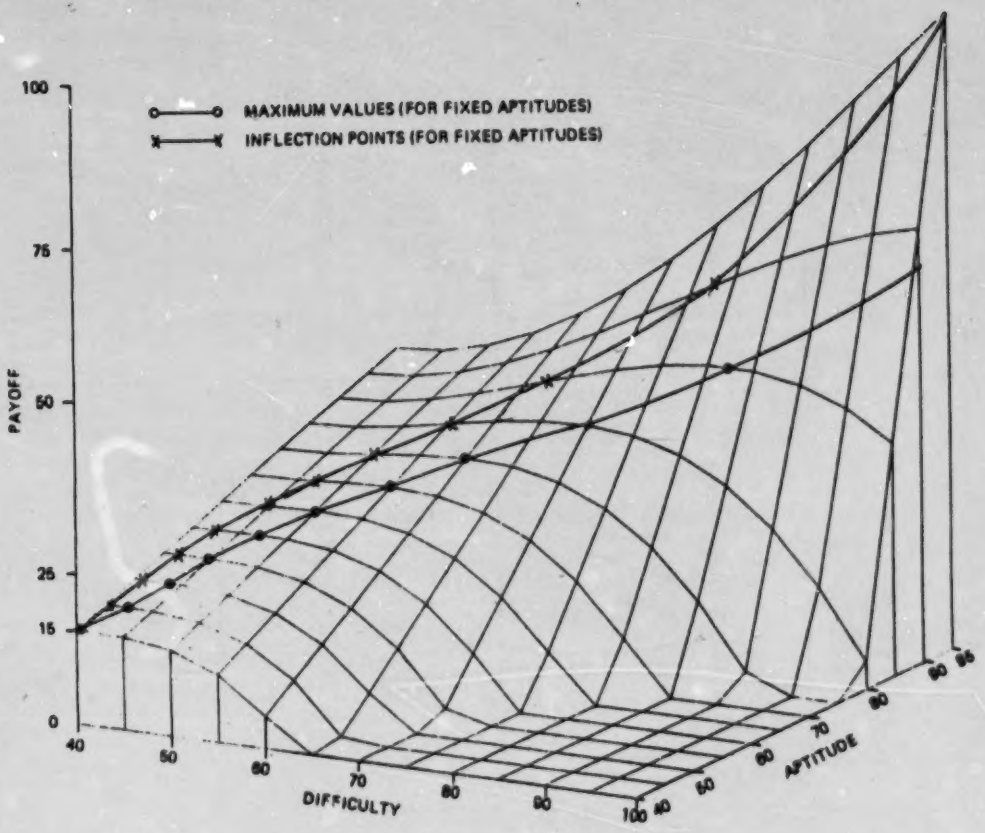


Figure 22. **Three-dimensional view of Model 3.**

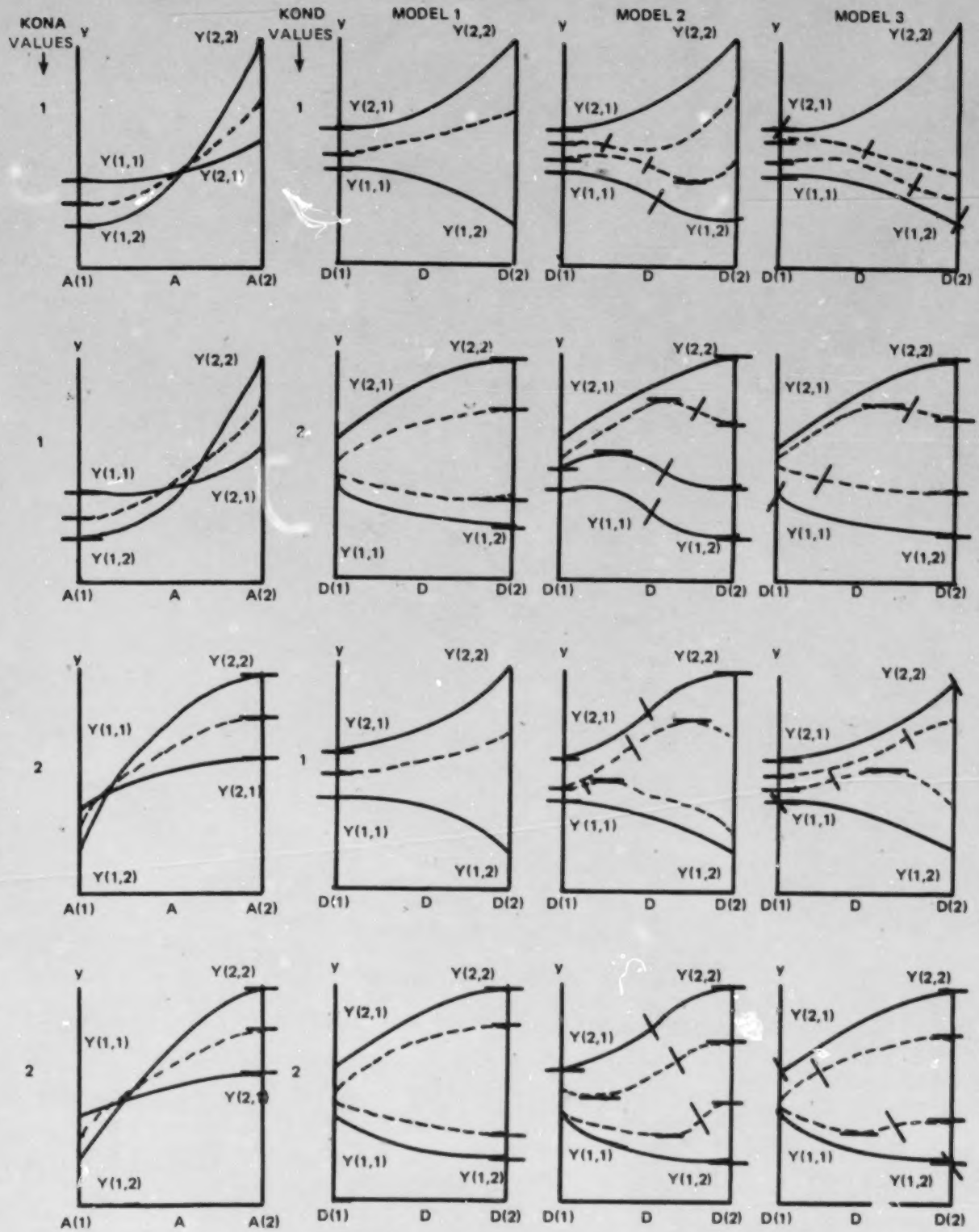


Figure 23. **Combining and comparing Models 1, 2, and 3.**

3. Y = 50 at Rank = 1 (high) and Preference = 1 (low); i.e., at A(1), D(1)

This requires

$$Y(1,1) = 50$$

4. Y = 25 at Rank = 15 (low) and Preference = 9 (high); i.e., at A(2), D(2). Then we set

$$Y(2,2) = 25$$

5. The values of Y will always be decreasing as values of Rank and Preference move away from Rank = 1 and Preference = 9. This implies the case of Model 1.

6. Near the highest value Y = 100 at Rank = 1 and Preference = 9 the Y values should decrease rapidly as Rank (variable A) increases for fixed Preference. The values will decrease slowly as Rank (A) approaches 15. This means that variable A will have a rapid change near A(1) and a slow change near A(2). This means that we should set

$$KONA = 2$$

7. Near the highest value Y = 100 at Rank = 1, Preference = 9 the Y values should decrease slowly as Preference (variable D) decreases for fixed Rank. The values will decrease rapidly as Preference (D) approaches the lowest value, 1. This means that variable D will have a rapid change near D(1) and a slow change near D(2). This requires the parameter setting

$$KOND = 2$$

8. The changes in Y values for Ranks near Rank = 15 should be slower (flatter) than the changes in Y values for Preferences near Preference = 9. Or another way to say this is that changes in Y values for Ranks near Rank = 1 should be faster (steeper) than the changes in Y values for Preferences near Preference = 1. This implies that AEXP > DEXP. To express these statements, values for the parameters were set to

$$AEXP = 4$$

$$DEXP = 2$$

The output of this policy specification is shown as Table 14.

Figure 24 shows a graphical representation of the function.

This policy-specified model was used to determine the probabilities for simulating the choice of an applicant. The Y values for all 15 jobs were determined from the policy-specified function. Then the probability of selection was given by

$$\text{Probability for Job } i = Y_i / \sum_{i=1}^{15} Y_i$$

The entire process of policy-specifying this model required less than one-half day.

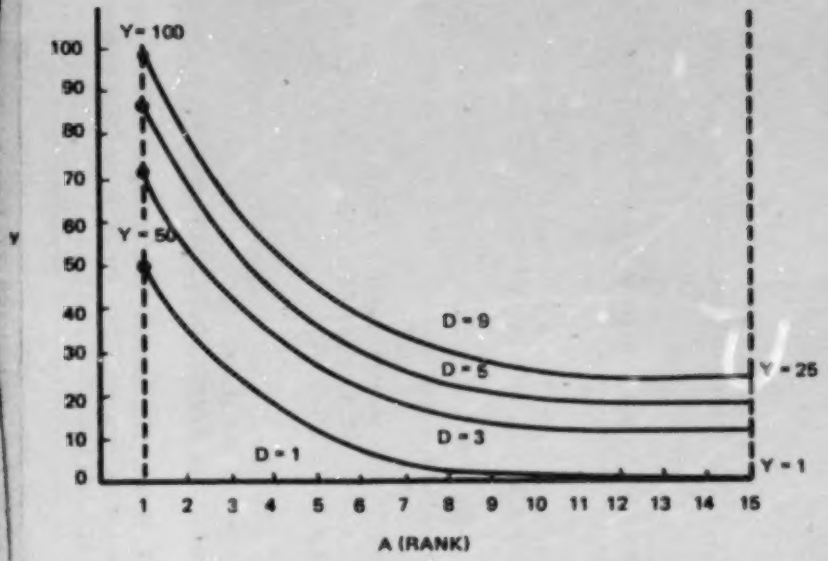
Table 14. Y (Tendency to Choose a Job) = Function of Rank Order on List and Preference Rating

```
MODEL=1

Y(1,1)=  50 Y(2,1)=   1 Y(1,2)= 100 Y(2,2)=  25
AEXP= 4 DEXP= 2 KONA= 2 KOND= 2
A(1)=  1 A(2)= 15 D(1)=  1 D(2)=  9
KSTARA=  1 KSTOPA= 15 KINCA=  1
KSTARD=  1 KSTOPD=  9 KINCD=  1

Y =   25 + [< .1952-02>*(A -  15)** 4] + [<-.3750+00>*(D -   9)** 2]
         + [<-.1058-04>*(A -  15)** 4]*(D -   9)** 2]]
```

(RANK) A	D (PREFERENCE) 1	2	3	4	5	6	7	8	9
1	50	62	72	80	88	93	97	99	100
2	37	48	56	64	70	75	78	80	81
3	27	36	44	51	56	60	63	65	65
4	20	28	35	40	45	49	51	53	54
5	14	21	27	33	37	40	43	44	45
6	9	16	22	27	31	34	36	37	38
7	6	12	18	23	26	29	31	33	33
8	4	10	15	20	23	26	28	29	30
9	3	8	14	18	21	24	26	27	28
10	2	8	12	17	20	23	25	26	26
11	1	7	12	16	19	22	24	25	25
12	1	7	12	16	19	22	24	25	25
13	1	7	12	16	19	22	24	25	25
14	1	7	12	16	19	22	24	25	25
15	1	7	12	16	19	22	24	25	25



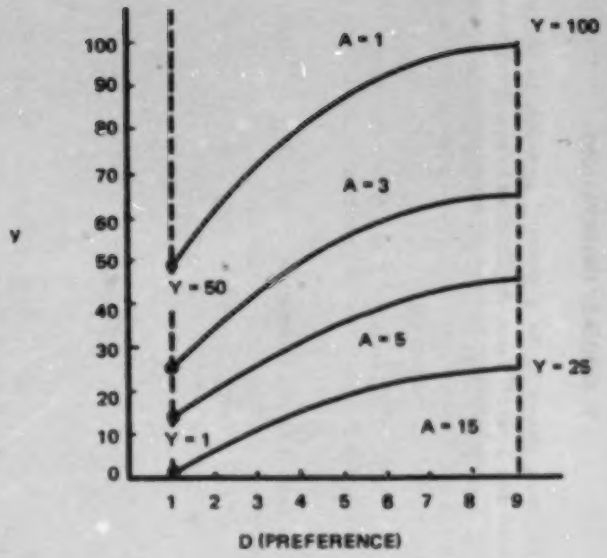


Figure 24. Y (tendency to choose) = function of A (rank) at preferences of 9, 5, 3, 1.
Y (tendency to choose) = function of D (preference) at ranks of 1, 3, 5, 15.

V. SUMMARY OBSERVATIONS

Policy-specifying has been shown to be a practical approach to obtaining implicit weights for information to be combined into a payoff value. Applications to new situations will continue to reveal the power of these models. Policy-specifying, combined with policy-capturing can provide a useful policy-development system. An on-line computer-based policy-development system should be implemented so that wide-spread use can be made of the process.

The policy-specifying models developed in this paper should be extended and improved to allow for easier use by the policy maker. The two-predictor models can be modified slightly to increase the power of the present system.

When it is desired to interact several variables, the two-variable models can be used repeatedly. For example, it might be appropriate to combine the Aptitude-Difficulty component—described in Example 1—in an interactive way with the Time-Fill component—described in Example 2. The resulting payoff generator would express a four-variable interaction among Aptitude, Difficulty, Time and Fill.

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*APPENDIX A:** MODEL DEVELOPMENT FOR EXAMPLE 1

Value to Air Force as Function of Aptitude and Job Difficulty

The policy of Example 1 will start with a polynomial of degree 1 in A (Aptitude) and degree 3 in D (Difficulty). When policy-specifying uses a polynomial form, great simplification is obtained by starting with a model in which the variables are expressed as deviations from a well-chosen constant. The constant is chosen according to the properties specified by the policy.

The starting model is

$$Y = B(0,0) + B(0,1) * (D - DK) + B(0,2) * (D - DK) ** 2 + B(0,3) * (D - DK) ** 3 + B(1,0) * (A - AK) + B(1,1) * (A - AK) * (D - DK) + B(1,2) * (A - AK) * (D - DK) * * 2 + B(1,3) * (A - AK) * (D - DK) * * 3$$

Since the policy seemed to imply that D-slopes would equal zero at D = 40 for all values of A, it was decided to choose DK = 40. Since A was only of degree equal one, the value of AK could be set to either of the critical values (AK = 40 or AK = 95) discussed in the specifying process. AK = 95 was the selected value. The model now becomes

$$Y = B(0,0) + B(0,1) * (D - 40) + B(0,2) * (D - 40) * * 2 + B(0,3) * (D - 40) ** 3 + B(1,0) * (A - 95) + B(1,1) * (A - 95) * (D - 40) + B(1,2) * (A - 95) * (D - 40) * * 2 + B(1,3) * (A - 95) * (D - 40) * * 3$$

There are eight unknown parameters, B's, to be determined from policy specifications.

Statement 1, Restrictions 1 - 2

D-slopes (i.e., slope of Y with respect to D) are equal zero at D = 40 for all values of A.

Letting Y1D = the first partial derivative of Y with respect to D

$$Y1D = B(0,1) + 2 * B(0,2) * (D - 40) + 3 * B(0,3) * (D - 40) * * 2 + B(1,1) * (A - 95) + 2 * B(1,2) * (A - 95) * (D - 40) + 3 * B(1,3) * (A - 95) * (D - 40) * * 2$$

At $D = 40$

$$Y1D = B(0,1) + B(1,1) * (A - 95)$$

Then setting

$$Y1D = 0 = B(0,1) + B(1,1) * (A - 95)$$

we observe that for the above statement to be true for all values of A it is necessary that

$$B(0,1) = 0 \quad \text{(Restriction 1)}$$
$$B(1,1) = 0 \quad \text{(Restriction 2)}$$

Then the model can be reduced to the six parameter form to simplify imposing the remaining restrictions.

---

*All capital letters are used consistently throughout the Appendixes. Note that * means "multiplication" and ** means "exponentiation."

$$Y = B(0,0) + B(0,2) * (D - 40) ** 2 + B(0,3) * (D - 40) ** 3 + B(1,0) * (A - 95) + B(1,2) * (A - 95) * (D - 40) ** 2 + B(1,3) * (A - 95) * (D - 40) ** 3$$

and

$$Y1D = 2 * B(0,2) * (D - 40) + 3 * B(0,3) * (D - 40) ** 2 + 2 * B(1,2) * (A - 95) * (D - 40) + 3 * B(1,3) * (A - 95) * (D - 40) ** 2$$

Statement 2, Restriction 3

D-slope equal zero at D = 100, A = 95

$$Y1D = 0 = 2 * B(0,2) * (100 - 40) + 3 * B(0,3) * (100 - 40) ** 2$$

$$0 = 2 * B(0,2) * 60 + 3 * B(0,3) * (60) ** 2$$

$$B(0,2) + 90 * B(0,3) = 0 \qquad \text{(Restriction 3)}$$

Statement 3, Restriction 4

The inflection point (second partial derivative of Y with respect to D = 0) occurs at A = 40, D = 40.

The second partial derivative of Y with respect to D is

$$Y2D = 2 * B(0,2) + 6 * B(0,3) * (D - 40) + 2 * B(1,2) * (A - 95) + 6 * B(1,3) * (A - 95) * (D - 40)$$

Evaluating at A = 40, D = 40 and setting Y2D = 0

$$2 * B(0,2) + 2 * B(1,2) * (40 - 95) = 0$$

$$B(0,2) + (-55) * B(1,2) = 0 \qquad \text{(Restriction 4)}$$

Statements 4-7, Restrictions 5-8

The values of Y were specified for four different combinations of A and D. The value of Y = -250 at A = 40, D = 100 was determined by experiment to make positive Y values begin just above the eligibility cut-off scores.

Then the requirements are

$$Y = 100 \text{ at } A = 95 \quad D = 100$$

$$Y = 35 \text{ at } A = 95 \quad D = 40$$

$$Y = 15 \text{ at } A = 40 \quad D = 40$$

$$Y = -250 \text{ at } A = 40 \quad D = 100$$

Evaluating the function at the above values we obtain

$$B(0,0) + B(0,2) * (60) ** 2 + B(0,3) * (60) ** 3 = 100 \qquad \text{(Restriction 5)}$$

$$B(0,0) = 35 \qquad \text{(Restriction 6)}$$

$$B(0,0) + B(1,0) * (-55) = 15 \qquad \text{(Restriction 7)}$$

$$B(0,0) + B(0,2) * (60) ** 2 + B(0,3) * (60) ** 3 + B(1,0) * (-55) + B(1,2) * (-55) * (60) ** 2 + B(1,3) * (-55) * (60) ** 3 = -250 \qquad \text{(Restriction 8)}$$

Restrictions 3-8 can now be summarized as follows:

	B(0,0)	B(0,2)	B(0,3)	B(1,0)	B(1,2)	B(1,3)	
							= 0
R-3		1	90				= 0
R-4		1			−55		= 100
R-5	1	$(60)^2$	$(60)^3$				= 35
R-6	1						= 15
R-7	1			−55			= −250
R-8	1	$(60)^2$	$(60)^3$	−55	$(-55)(60)^2$	$(-55)(60)^3$	

These six restrictions can be imposed as follows:

From R-6

B(0,0) = 35

From R-7

35 - 55 * B(1,0) = 15 and B(1,0) = .3636

From R-3 and R-5

(R-3) B(0,2) + 90 * B(0,3) = 0

(R-5) B(0,0) + B(0,2) * (60) * * 2 + B(0,3) * (60) * * 3 = 100

Substituting B(0,0) = 35 in (R-5) and multiplying (R-3) by (60) * * 2

(R-5) B(0,2) * (60) * * 2 + B(0,3) * (60) * * 3 = 65

(R-3) B(0,2) * (60) * * 2 + B(0,3) * 90 * (60) * * 2 = 0

B(0,3) = 65/((60) * * 2) * (−30)

B(0,3) = (−13)/(21600) = −.0006019

Substituting in R-3

(R-3) B(0,2) + 90 * B(0,3) = 0

B(0,2) = ((−90) * (−13))/(21600) = 65/1200

B(0,2) = 13/240 = .05417

Substituting in R-4

(R-4) B(0,2) + (−55) * B(1,2) = 0

B(1,2) = (−13)/(240 * (−55))

B(1,2) = .0009848

Substituting in R-8

B(1,3) = (−135)/(−55 * (60) * * 3)

B(1,3) = 1/88000 = .00001136

Then the policy-specified model of Example 1 is

$$Y = 35 + .05417 * (D - 40) * * 2 - .0006019 * (D - 40) * * 3 + .3636 * (A - 95) + .0009848 * (A - 95) * (D - 40) * * 2 + .00001136 * (A - 95) * (D - 40) * * 3$$

APPENDIX B: MODEL DEVELOPMENT FOR EXAMPLE 2

Value to Air Force as Function of Time Used and Fraction of Fill

The policy of Example 2 starts with a polynomial of degree 1 for T (Time Used) and for F (Fraction of Fill).

The general model is

$$Y = B(0,0) + B(0,1) * (F - FK) + B(1,0) * (T - TK) + B(1,1) * (T - TK) * (F - FK)$$

Since the policy desires specific Y values when F = 0 and when T = 0, it is convenient to set FK = 0 and TK = 0. Then the model becomes

$$Y = B(0,0) + B(0,1) * F + B(1,0) * T + B(1,1) * T * F$$

There are only four unknown parameters to be specified; therefore, the four critical Y values are sufficient to determine the unknown B's.

Statements 1-4, Restrictions 1-4

Y = 100	T = 180	F = 0
Y = 0	T = 0	F = 1.0
Y = K	T = 0	F = 0
Y = K	T = 180	F = 1.0

These statements are then used to obtain the restrictions as follows:

R-1 $B(0,0) + B(1,0) * 180 = 100$

R-2 $B(0,0) + B(0,1) = 0$

R-3 $B(0,0) = K$

R-4 $B(0,0) + B(0,1) + B(1,0) * 180 + B(1,1) * 180 = K$

Then we have from R-3

$$B(0,0) = K$$

from R-2

$$B(0,1) = -K$$

from R-1

$$B(1,0) = (100 - K)/180$$

from R-4

$$B(1,1) = (2 * K - 100)/180$$

Then the model becomes

$$Y = K + (-K) * F + ((100 - K)/180) * T + ((2 * K - 100)/180) * T * F$$

and when K = 25

$$Y = 25 + (-25 * F) + (.4167 * T) + (-.2778 * T * F)$$

and when K = 75

$$Y = 75 + (-75 * F) + (.1389 * T) + (.2778 * T * F)$$

APPENDIX C: MODEL DEVELOPMENT FOR EXAMPLE 3

Value to Air Force as Function of Fill and Known Goal

This model can be developed from observing the sketch in Figure C-1 of the policy statements for three goals.

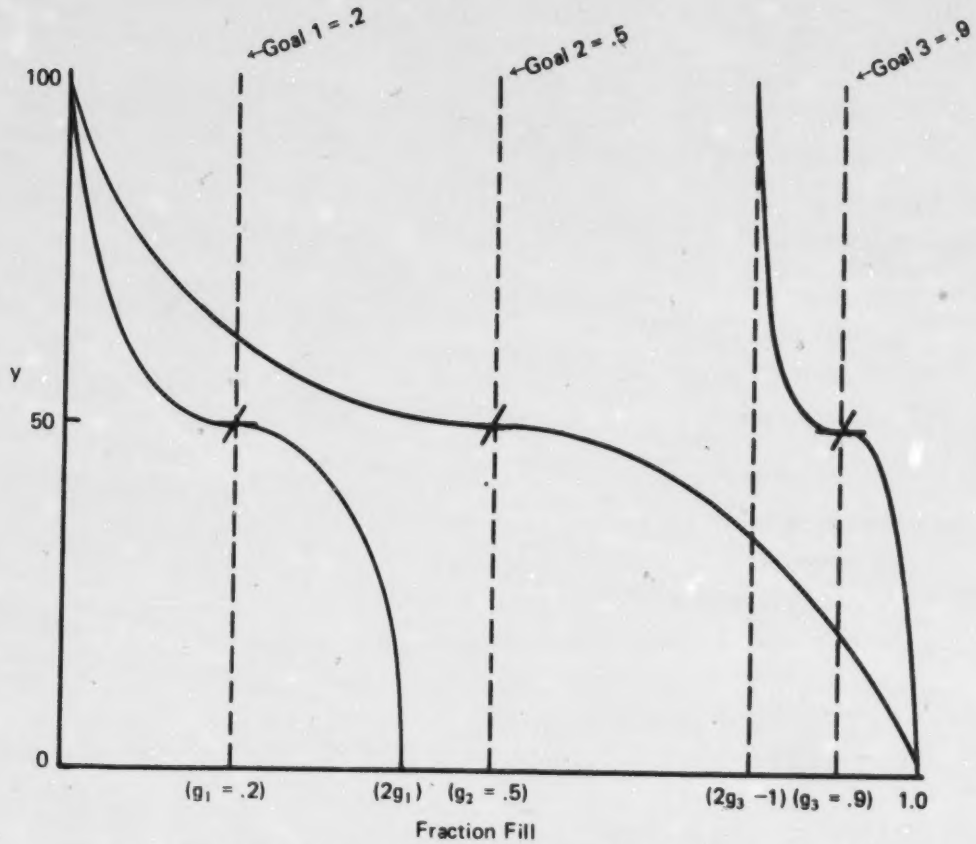

Figure C-1. Sketch of policy from example 3 showing the function for three different goals.

The model starts with a general polynomial of degree 3 for variable F.

$$Y = B(0) + B(1) * (F - FK) + B(2) * (F - FK) * * 2 + B(3) * (F - FK) * * 3$$

Since the critical values of Y are specified when F = the known goal (G), we will let the value of FK = G. Then our model is

$$Y = B(0) + B(1) * (F - G) + B(2) * (F - G) * * 2 + B(3) * (F - G) * * 3$$

There are four parameters to be determined at this time.

Statements 1 and 2, Restrictions 1 and 2

We would like the slope of Y with respect to F to be zero when F = G and the inflection point to occur when F = G.

Therefore we write the first derivative

$$Y1F = B(1) + 2 * B(2) * (F - G) + 3 * B(3) * (F - G) * * 2$$

and the second derivative

$$Y2F = 2 * B(2) + 6 * B(3) * (F - G)$$

Then we have the restrictions that

$Y1F = 0$ at $F = G$

$Y2F = 0$ at $F = G$

therefore

$$B(1) = 0 = Y1F$$

$$B(2) = 0 = Y2F$$

Then our first two restrictions imply $B(1) = 0$ and $B(2) = 0$ so the model becomes

$$Y = B(0) + B(3) * (F - G) * * 3$$

Statements 3 and 4, Restrictions 3 and 4

First consider $0 < G \leq .5$

Then whenever F = G, we want Y = 50 and when F = 2G, Y = 0.

Then we impose these two restrictions and obtain:

$B(0) = 50$

$B(0) + B(3) * (G) * * 3 = 0$

$$B(3) = -50/G * * 3$$

Then we have the model for $0 < G \leq .5$

$$Y = 50 + (50/G * * 3) * (G - F) * * 3$$

For the special cases

$G = 0$ $F \neq 0$ then $Y = 0$

$G = 0$ $F = 0$ then $Y = 50$

and when $0 < G \leq .5$ and $2G < F \leq 1$

$Y = 0$

Next consider the range $.5 < G < 1$

Whenever $F = G$, we want $Y = 50$ and when

$F = 2G - 1$, we want $Y = 100$.

Then imposing these two restrictions we have

$B(0) = 50$

$B(0) + B(3) * (G - 1) ** 3 = 100$

$B(3) = 50/(G - 1) ** 3$

Then we write the model for $.5 < G < 1$ and $(2G - 1) \leq F \leq 1$

$Y = 50 + (50/(1 - G) ** 3) * (G - F) ** 3$

For the special cases

$G = 1$ $F \neq 1$ then $Y = 100$

$G = 1$ $F = 1$ then $Y = 50$

and when $0 \leq F < (2G - 1)$

$Y = 100$

It is sometimes convenient to create a default option which maintains an on-target value of $Y = 50$ for all values of F. This is accomplished by setting $Y = 50$ when $G = 0$ or $G = 1$. If this procedure is used, it is necessary to assign G equal a small positive value near zero to represent a desired goal of zero, or to assign G equal a value slightly less than 1 to represent a desired goal of one.

APPENDIX D: MODEL DEVELOPMENT FOR EXAMPLE 4

Value to Air Force as a Function of Several Components

When several components are to be combined into a single composite, a policy maker may wish to control the "relative amount" that each component contributes to the composite. One approach to this is to convert the range of each variable into a fractional part of the composite.

Start with a model of N components of the form

$$Y = B(0) + B(1) * (X(1) - K(1)) + \ldots + B(I) * (X(I) - K(I)) + \ldots + B(N) * (X(N) - K(N))$$

There are N + 1 parameters to be determined. Since we wish to make statements of policy at high (H(I)) or low (L(I)) values of each variable X(I), we can arbitrarily set K(I) = L(I).

Then the starting model becomes

$$Y = B(0) + B(1) * (X(1) - L(1)) + \ldots + B(I) * (X(I) - L(I)) + \ldots + B(N) * (X(N) - L(N))$$

We would like the following conditions to be satisfied:

Let

HC = high value of composite

LC = low value of composite

F(I) = fraction of composite to be used by X(I)

Then we want

$$Y = LC \text{ when } X(I) = L(I) \qquad I = 1, \ldots, N$$

Substituting in the model to express this restriction we obtain

$$B(0) = LC$$

And when X(I) = H(I) and X(J) = L(J) for $J \neq I$

$$Y \text{ equals } LC + F(I)\,(HC - LC)$$

This implies

$$LC + B(I) * (H(I) - L(I)) = LC + F(I) * (HC - LC) \qquad I = 1, \ldots, N$$

Then we have

$$B(I) = (F(I) * (HC - LC))/(H(I) - L(I)) \qquad I = 1, \ldots, N$$

Substituting these values for B(0) and B(I) gives

$$Y = LC + ((F(1) * (HC - LC))/(H(1) - L(1))) * (X(1) - L(1)) + \ldots + ((F(I) * (HC - LC))/(H(I) - L(I))) * (X(I) - L(I)) + \ldots + (F(N) * (HC - LC))/(H(N) - L(N))) * (X(N) - L(N))$$

APPENDIX E: MODEL DEVELOPMENT FOR GENERALIZED MODEL 1

The major restrictions for Model 1 are that there be one and only one value of variable A (call it A(KONA)) for which the A-slope equals 0 at every value of D; and that there be one and only one value of variable D (call it D(KOND)) for which the D-slope equals 0 at every value of A.

The polynomial form of degrees AEXP and DEXP in A and D that has the above properties is

$$Y = B(1) + B(2) * (A - A(KONA)) ** AEXP + B(3) * (D - D(KOND)) ** DEXP + B(4) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** DEXP)$$

The four unknown values B(1), . . . , B(4) can be determined by policy-specifying the Y values at four critical combinations of A and D.

The A values for control are designated

A(KONA) and A(KONACH)

and the D control values are

D(KOND) and D(KONDCH)

The policy statements give explicit values of Y for all four combinations of A and D control values. These four Y values are named

Y(KONA, KOND)

Y(KONA, KONDCH)

Y(KONACH, KOND)

Y(KONACH, KONDCH)

Then the four restrictions imposed on the model are

(R-1) $B(1) = Y(KONA, KOND)$

(R-2) $B(1) + B(3) * (D(KONDCH) - D(KOND)) ** DEXP = Y(KONA, KONDCH)$

(R-3) $B(1) + B(2) * (A(KONACH) - A(KONA)) ** AEXP = Y(KONACH, KOND)$

(R-4) $B(1) + B(2) * (A(KONACH) - A(KONA)) ** AEXP + B(3) * (D(KONDCH) - D(KOND)) ** DEXP + B(4) * ((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP) = Y(KONACH, KONDCH)$

Solving these four restrictions we obtain from R-1

$$B(1) = Y(KONA, KOND)$$

from R-2

$$B(3) = (Y(KONA, KONDCH) - Y(KONA, KOND))/((D(KONDCH) - D(KOND)) ** DEXP)$$

from R-3

$$B(2) = (Y(KONACH, KOND) - Y(KONA, KOND))/((A(KONACH) - A(KONA)) ** AEXP)$$

and from R-4

$$B(4) = (Y(KONA, KOND) - Y(KONA, KONDCH) - Y(KONACH, KOND) + Y(KONACH, KONDCH))/(((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP))$$

APPENDIX F: MODEL DEVELOPMENT FOR GENERALIZED MODEL 2

The major restrictions for Model 2 are that there be one and only one value of variable A (call it A(KONA)) for which the A-slope equals 0 at every value of D (the same as Model 1); but there should be two and only two values of variable D (one of which is D(KOND)) for which the D-slope equals 0 at every value of A.

The polynomial form of degrees AEXP and DEXP in A and D that has the above properties is

```
Y = B(1) + B(2) * (A - A(KONA)) ** AEXP
         + B(3) * (D - D(KOND)) ** (DEXP - 1)
         + B(4) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** DEXP)
         + B(5) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** (DEXP - 1 ))
         + B(6) * (D - D(KOND)) ** DEXP
```

In order to control the movement of a maximum (or minimum) ridge (or valley) we will require two special restrictions on the D-slopes.

Statements 1 and 2; Restrictions 1 and 2

The starting model above already has D-slope equal zero at D(KOND). To control the ridge (or valley) movements we will require that

(1) When A = A(KONA), the D-slope = 0 at D = D(KONDCH) and that

(2) When A = A(KONACH), the D-slope = 0 only at D = D(KOND). One sketch of relationships that have these conditions is presented in Figure F-1.

Then we write the expression for first partial derivative of Y with respect to D.

```
Y1D = B(3) * (DEXP - 1) * (D - D(KOND)) ** (DEXP - 2)
    + B(4) * DEXP * ((A - A(KONA)) ** AEXP) * (D - D(KOND)) ** (DEXP - 1)
    + B(5) * (DEXP - 1) * ((A - A(KONA)) ** AEXP) * (D - D(KOND)) ** (DEXP - 2)
    + B(6) * DEXP * (D - D(KOND)) ** (DEXP - 1)
```

then the two restrictions become,

when A = A(KONA) and D = D(KONDCH), Y1D = 0 or

```
(R-1)  B(3) * (DEXP - 1) * (D(KONDCH) - D(KOND)) ** (DEXP - 2) + B(6) * DEXP * (D(KONDCH) -
       - D(KOND)) ** (DEXP - 1) = 0
```

and when A = A(KONACH) the D-slope = 0 only at D = D(KOND)

Rewriting and factoring (D - D(KOND)) ** (DEXP - 2)

```
Y1D = ((D - D(KOND)) ** (DEXP - 2)) * [B(3) * (DEXP - 1) + B(4) * DEXP * ((A(KONACH) - A(KONA) **
      AEXP) * (D - D(KOND)) + B(5) * (DEXP - 1) * (A(KONACH) - A(KONA)) ** AEXP
      + B(6) * DEXP * (D - D(KOND))]
```

Then we observe that the above expression is guaranteed to be zero only at D = D(KOND) by requiring that

```
(R-2)  D(3) * (DEXP - 1) + B(5) * (DEXP - 1) * (A(KONACH) - A(KONA)) ** AEXP = 0
```

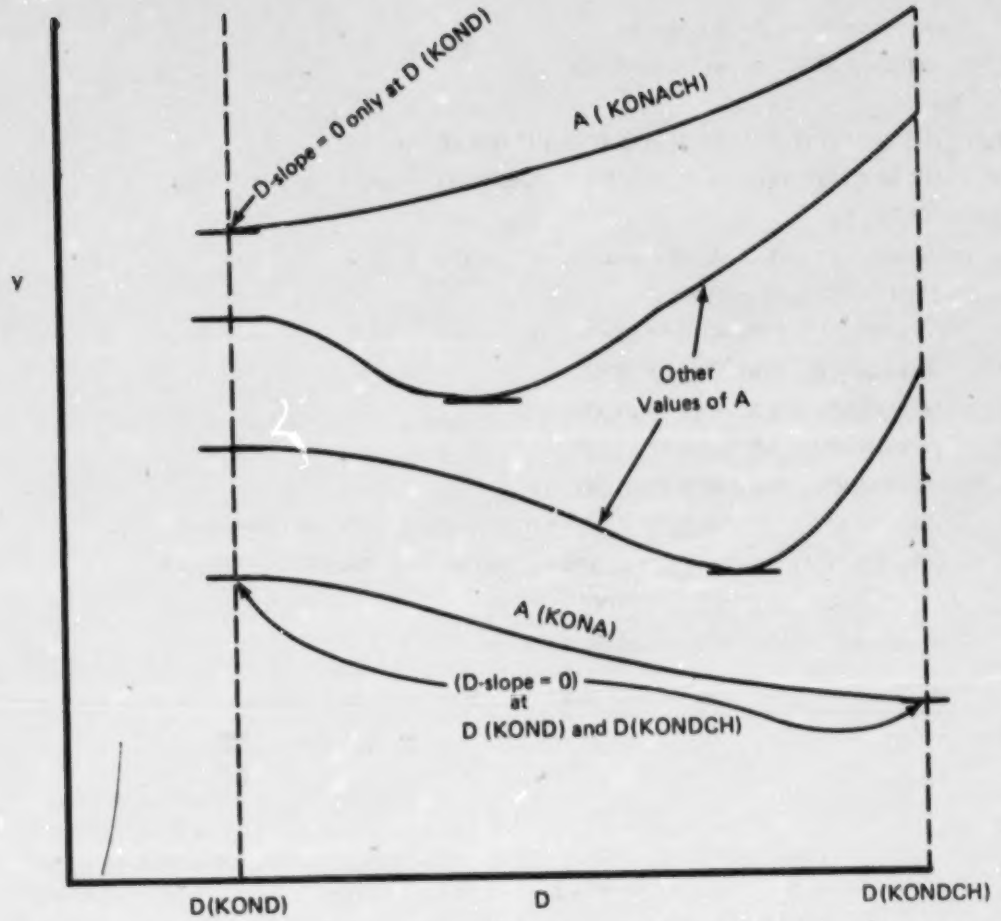

Figure F-1. **Sketch of conditions for D-slopes in generalized Model 2.**

Statements 3-6; Restrictions 3-6

The remaining four restrictions are provided by specifying (as in Model 1)

Y(KONA, KOND)

Y(KONA, KONDCH)

Y(KONACH, KOND)

Y(KONACH, KONDCH)

The four restrictions are

(R-3) B(1) = Y (KONA, KOND)

(R-4) B(1) + B(3) * (D(KONDCH) - D(KOND)) ** (DEXP - 1) + B(6) * (D(KONDCH) - D(KOND)) ** DEXP = Y(KONA, KONDCH)

(R-5) B(1) + B(2) * (A(KONACH) - A(KONA)) ** AEXP = Y(KONACH, KOND)

(R-6) B(1) + B(2) * (A(KONACH) - A(KONA)) ** AEXP + B(3) * (D(KONDCH) - D(KOND)) ** (DEXP -1)
+ B(4) * ((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP)
+ B(5) * ((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** (DEXP - 1))
+ B(6) * (D(KONDCH) - D(KOND) ** DEXP = Y(KONACH, KONDCH)

Now to simplify the development let

C(I,J) = coefficient of B(J) in the Ith restriction

then

C(1,3) = (DEXP - 1) * (D(KONDCH) - D(KOND)) ** (DEXP - 2)

C(1,6) = (DEXP * (D(KONDCH) - D(KOND)) ** (DEXP - 1)

C(2,3) = (DEXP - 1)

C(2,5) = (DEXP - 1) * (A(KONACH) - A(KONA)) ** AEXP

C(3,1) = C(4,1) = C(5,1) = C(6,1) = 1

C(4,3) = (D(KONDCH) - D(KOND)) ** (DEXP - 1)

C(4,6) = (D(KONDCH) - D(KOND)) ** DEXP

C(5,2) = (A(KONACH) - A(KONA)) ** AEXP

C(6,2) = (A(KONACH) - A(KONA)) ** AEXP

C(6,3) = (D(KONDCH) - D(KOND)) ** (DEXP - 1)

C(6,4) = ((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP

C(6,5) = ((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** (DEXP - 1)

C(6,6) = (D(KONDCH) - D(KOND)) ** DEXP

Now the restrictions can be summarized as follows:

	B(1)	B(2)	B(3)	B(4)	B(5)	B(6)	
R-1			C(1,3)			C(1,6)	= 0
R-2			C(2,3)		C(2,5)		= 0
R-3	1						= Y(KONA, KOND)
R-4	1		C(4,3)			C(4,6)	= Y(KONA, KONDCH)
R-5	1	C(5,2)					= Y(KONACH, KOND)
R-6	1	C(6,2)	C(6,3)	C(6,4)	C(6,5)	C(6,6)	= Y(KONACH, KONDCH)

From R-3 we can obtain

B(1) = Y(KONA, KOND)

from R-5

B(2) = Y(KONCH, KOND) - Y(KONA, KOND))/C(5,2)

B(2) = (Y(KONCH, KOND) - Y (KONA, KOND))/((A(KONACH) - A(KONA)) ** AEXP)

from R-1 and R-4

B(3) * C(1,3) + B(6) * C(1,6) = 0

B(1) + B(3) * C(4,3) + B(6) * C(4,6) = Y(KONA, KONDCH)

Solving first for B(3) gives

B(3) = (C(1,6) * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((C(4,3) * C(1,6)) - (C(1,3) * C(4,6)))

and after substitution

$$B(3) = (DEXP * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((D(KONDCH) - D(KOND)) ** (DEXP - 1))$$

Using R-2

$$B(3) * C(2,3) + B(5) * C(2,5) = 0$$

and

$$B(5) = (-B(3) * C(2,3))/C(2,5)$$

$$B(5) = (-B(3))/(A(KONACH) - A(KONA)) ** AEXP$$

Using R-1

$$B(3) * C(1,3) + B(6) * C(1,6) = 0$$

and

$$B(6) = (-B(3) * C(1,3))/C(1,6)$$

$$B(6) = ((-B(3)) * (DEXP - 1))/(DEXP * (D(KONDCH) - D(KOND)))$$

Using R-6

$$B(1) + B(2) * C(6,2) + B(3) * C(6,3) + B(4) * C(6,4) + B(5) * C(6,5) + B(6) * C(6,6) = Y(KONACH, KONDCH)$$

Solving for B(4) gives

$$B(4) = (Y(KONACH, KONDCH) - Y(KONACH, KOND) + ((DEXP - 1) * (Y(KONA, KONDCH) - Y(KONA, KOND))))/(((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP))$$

APPENDIX G: MODEL DEVELOPMENT FOR GENERALIZED MODEL 3

The only difference between Model 2 and this model is that restrictions 1 and 2 in this model will control inflection points rather than D-slope = 0.

The polynomial form to start is as in Model 2

```
Y = B(1) + B(2) * (A - A(KONA)) ** AEXP
         + B(3) * (D - D(KOND)) ** (DEXP - 1)
         + B(4) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** DEXP)
         + B(5) * ((A - A(KONA)) ** AEXP) * (DD - D(KOND)) ** (DEXP - 1))
         + B(6) * (D - D(KOND)) ** DEXP
```

Statements 1 and 2; Restrictions 1 and 2

To control the inflection points we will write the expression for second partial derivative of Y with respect to D.

```
Y2D = B(3) * (DEXP - 2) * (DEXP - 1) * (D - D(KOND)) ** (DEXP - 3)
    + B(4) * (DEXP - 1) * (DEXP) * ((A - A(KONA)) ** AEXP) * (D - D(KOND)) ** (DEXP - 2)
    + B(5) * (DEXP - 2) * (DEXP - 1) * ((A - A(KONA)) ** AEXP) * (D - D(KOND)) ** (DEXP - 2)
    + B(6) * (DEXP - 1) * (DEXP) * (D - D(KOND)) ** (DEXP - 2)
```

Then the restriction for inflection points control are

When A = A(KONA) and D = D(KONDCH), Y2D = 0 or

```
(R-1)  B(3) * (DEXP - 2) * (DEXP - 1) * (D(KONDCH) - D(KOND)) ** (DEXP - 3) + B(6) * (DEXP - 1)
                  * (DEXP) * (D(KONDCH) - D(KOND)) ** (DEXP - 2) = 0
```

and when A = A(KONACH), Y2D = 0 only at D = D(KOND)

Rewriting and factoring (D - D(KOND)) ** (DEXP - 3)

```
Y2D = ((D - D(KOND)) ** (DEXP - 3)) * (B(3) * (DEXP - 2) * (DEXP - 1)
    + B(4) * (DEXP - 1) * DEXP * ((A(KONACH) - A(KONA)) ** AEXP) * (D - D(KOND))
    + B(5) * (DEXP - 2) * (DEXP - 1) * (A(KONACH) - A(KONA) ** AEXP
    + B(6) * (DEXP - 1) * (DEXP * (D - D(KOND))
```

Then we observe that the above expression is guaranteed to be zero only at D = D(KOND) by requiring that

```
(R-2)  B(3) * (DEXP - 2) * (DEXP - 1) + B(5) * (DEXP - 2) * (DEXP - 1) * (A(KONACH) - A(KONA))
           ** AEXP = 0
```

Then the remaining four restrictions are the same as in Model 2.

Now we can let

D(1,3) = (DEXP - 2) * (DEXP - 1) * (D(KONDCH) - D(KOND)) ** (DEXP - 3)

But noticing the value of C(1,3) and C(1,6) in Model 2 (Appendix F)

D(1,3) = C(1,3) * ((DEXP - 2)/(D(KONDCH) - D(KOND)))

and

D(1,6) = (DEXP - 1) * (DEXP) * (D(KONDCH) - D(KOND)) ** (DEXP - 2)

D(1,6) = C(1,6) * ((DEXP - 1)/(D(KONDCH) - D(KOND)))

and

D(2,3) = (DEXP - 2) * (DEXP - 1)

D(2,3) = C(2,3) * (DEXP - 2)

Furthermore,

D(2,5) = (DEXP - 2) * (DEXP - 1) * (A(KONACH) - A(KONA) ** AEXP

D(2,5) = C(2,5) * (DEXP - 2)

Then the solutions for B(1), B(2), and B(5) are the same as for Model 2 and we have new values for B(3), B(4), and B(6)

B(3) = (D(1,6) * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((C(4,3) * D(1,6)) - (D(1,3) * C(4,6)))

and after substitution

B(3) = ((DEXP/2) * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((D(KONDCH) - D(KOND)) ** (DEXP - 1)

and

B(6) = ((-B(3)) * D(1,3))/D(1,6)

B(6) = ((-B3)) * (DEXP - 2))/(DEXP * (D(KONDCH) - D(KOND)))

Using restriction (R-6) gives

B(4) = (Y(KONACH), KONDCH) - Y(KONACH, KOND) + (((DEXP - 2)/2) * (Y(KONA, KONDCH) - Y(KONA, KOND))))/(((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP)

APPENDIX H: COMBINING MODEL 2 AND MODEL 3

By observing the close similarity between Model 2 and Model 3 we can insert a new parameter

MOD = 2 if Model 2
or 3 if Model 3

Then the coefficients for Model 2 or Model 3 can be determined by setting MOD = 2 or 3 in B(3), B(4), and B(6). B(1), B(2), and B(5) are the same in both models.

B(1) = Y(KONA, KOND)

B(2) = (Y(KONACH, KOND) - Y(KONA, KOND))/((A(KONACH) - A(KONA) ** AEXP

B(3) = (DEXP/(MOD - 1)) * (Y(KONA, KONDCH) - Y(KONA, KOND))/((D(KONDCH) - D(KOND)) ** (DEXP - 1)

B(4) = (Y(KONACH, KONDCH) - Y(KONACH, KOND) + (((DEXP - MOD + 1/(MOD - 1)) * (Y(KONA, KONDCH) - Y(KONA, KOND))))/(((A(KONACH) - A(KONA) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP)

B(5) = (-B(3))/(A(KONACH) - A(KONA)) ** AEXP

B(6) = ((-B(3)) * (DEXP - MOD + 1))/(DEXP * (D(KONDCH) - D(KOND)))